AP® Physics C: Electricity and Magnetism
2003 Sample Student Responses

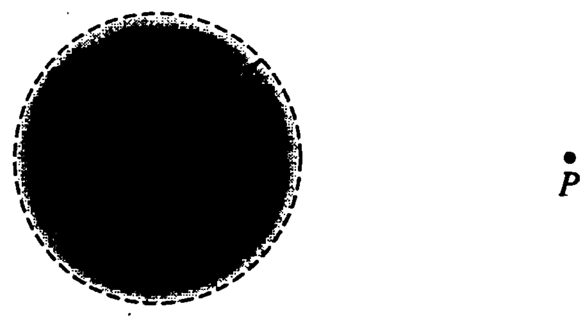
PHYSICS C

Section II, ELECTRICITY AND MAGNETISM

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.



E&M. 1.

A spherical cloud of charge of radius R contains a total charge $+Q$ with a nonuniform volume charge density that varies according to the equation

$$\rho(r) = \rho_0\left(1 - \frac{r}{R}\right) \text{ for } r \leq R \text{ and}$$
$$\rho = 0 \text{ for } r > R,$$

where r is the distance from the center of the cloud. Express all algebraic answers in terms of Q, R, and fundamental constants.

(a) Determine the following as a function of r for $r > R$.

i. The magnitude E of the electric field

$\oint E \cdot dA = \frac{q}{\epsilon_0}$

$E(4\pi r^2) = \frac{Q}{\epsilon_0}$ $\quad E = \frac{Q}{4\pi\epsilon_0 r^2}$

ii. The electric potential V

A spherical distribution acts as a point charge so

$V = \frac{Q}{4\pi\epsilon_0 r}$

(b) A proton is placed at point P shown above and released. Describe its motion for a long time after its release.

The proton experiences a force to the right from the electric field. The field strength decreases with distance, so the acceleration of the proton decreases with time until it reaches a final constant speed.

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(c) An electron of charge magnitude e is now placed at point P, which is a distance r from the center of the sphere, and released. Determine the kinetic energy of the electron as a function of r as it strikes the cloud.

Energy conserved

$K_i + U_i = K_f + U_f$

$0 + qV_1 = K_f + U_f$

$qV_1 - qV_2 = K_f$

$$-\frac{eQ}{4\pi\epsilon_0 r} + \frac{eQ}{4\pi\epsilon_0 R} = K(r) = \frac{eQ}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right)$$

(d) Derive an expression for ρ_0.

$dq = \rho(r)\,dV \qquad dV = 4\pi r^2 dr$

$$\int dq = \int_0^R \rho_0\left(1 - \frac{r}{R}\right)4\pi r^2 dr$$

$$Q = 4\pi\rho_0 \int_0^R r^2\left(1 - \frac{r}{R}\right)dr$$

$$Q = 4\pi\rho_0 R^3\left(\frac{1}{3} - \frac{1}{4}\right)$$

$$\rho_0 = \frac{Q}{4\pi R^3\left(\frac{1}{12}\right)} = \frac{3Q}{\pi R^3}$$

(e) Determine the magnitude E of the electric field as a function of r for $r \leq R$.

$$\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0} \rightarrow \quad dq = \rho(r)\,dV$$

$$dq_{enc} = \rho(r)(4\pi r^2 dr)$$

$$q_{enc} = \int_0^r \rho_0\left(1 - \frac{r}{R}\right)(4\pi r^2)dr$$

$$q_{enc} = 4\pi\rho_0 \int_0^r \left(1 - \frac{r}{R}\right)r^2 dr = 4\pi\rho_0 \int_0^r \left(r^2 - \frac{r^3}{R}\right)dr$$

$$q_{enc} = 4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^r$$

$$q_{enc} = 4\pi\rho_0 r^3\left(\frac{1}{3} - \frac{r}{4R}\right)$$

$$(4\pi r^2)E = \frac{4\pi\rho_0 r^3\left(\frac{1}{3} - \frac{r}{4R}\right)}{\epsilon_0}$$

$$E = \frac{\rho_0 r\left(\frac{1}{3} - \frac{r}{4R}\right)}{\epsilon_0}$$

$$= \frac{3Qr}{\pi\epsilon_0 R^3}\left(\frac{1}{3} - \frac{r}{4R}\right)$$

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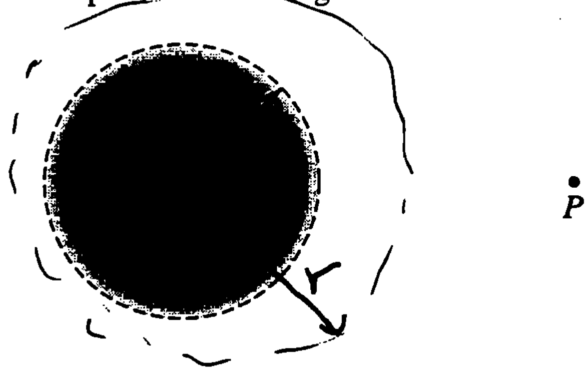

E&M. 1.

A spherical cloud of charge of radius R contains a total charge $+Q$ with a nonuniform volume charge density that varies according to the equation

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where r is the distance from the center of the cloud. Express all algebraic answers in terms of Q, R, and fundamental constants.

(a) Determine the following as a function of r for $r > R$.

i. The magnitude E of the electric field

$\int E \cdot dA = \frac{Q_{encl}}{\epsilon_0}$ $\quad E\int dA = \frac{Q_{encl}}{\epsilon_0}$ $\quad EA = \frac{Q}{\epsilon_0}$

$E(4\pi r^2) = \frac{Q}{\epsilon_0}$

$E = \frac{Q}{4\pi r^2 \epsilon_0}$

ii. The electric potential V

$E = -\frac{dV}{dr}$ $\quad \frac{-Q}{4\pi r^2 \epsilon_0}\,dr = dV$ $\quad \frac{-Q}{4\pi \epsilon_0}\int \frac{dr}{r^2} = V$

$V = \frac{-Q}{4\pi \epsilon_0}(-r^{-1})$ $\quad V = \frac{Q}{4\pi \epsilon_0 r}$

(b) A proton is placed at point P shown above and released. Describe its motion for a long time after its release.

the proton will accelerate away from the cloud along a straight line from the center of the cloud through point P. The acceleration decreases as the proton gets farther away, but the velocity increases

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(c) An electron of charge magnitude e is now placed at point P, which is a distance r from the center of the sphere, and released. Determine the kinetic energy of the electron as a function of r as it strikes the cloud.

$U_{E_i} = qV = \frac{eQ}{4\pi\epsilon_0 r}$

$U_{Ef} = \frac{eQ}{4\pi\epsilon_0 R}$

$U_{Ei} = U_{Ef} + K \qquad K = U_{Ei} - U_{Ef} = \frac{eQ}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right) = K(r)\ \text{J}$

(d) Derive an expression for ρ_0.

~~$Q = \rho V$~~

~~$Q = \frac{4}{3}\pi R^3 [\rho_0 ($~~

$q = \int \rho\, dV \qquad V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 dr$

$Q = \int_0^R [\rho_0(1 - \frac{r}{R})](4\pi r^2)dr$

$Q = \rho_0 \int_0^R \left(4\pi r^2 - \frac{4\pi r^3}{R}\right)dr$

$= \rho_0 4\pi \int_0^R \left(r^2 - \frac{r^3}{R}\right)dr$

$= \rho_0 4\pi \left(\frac{1}{3}R^3 - \frac{1}{4}\frac{R^4}{R}\right) = \rho_0 4\pi\left(\frac{1}{12}R^3\right)$

$Q = \frac{1}{3}\rho_0 \pi R^3 \qquad \rho_0 = 3Q/\pi R^3$

(e) Determine the magnitude E of the electric field as a function of r for $r \le R$.

$\int E \cdot dA = \frac{Q_{incl}}{\epsilon_0} \qquad EA = \frac{q}{\epsilon_0}$

$q = \rho_0 4\pi\left(\frac{1}{3}r^3 - \frac{1}{4}\frac{r^4}{R}\right)$

$= (3Q/\pi R^3)4\pi\left(\frac{1}{3}r^3 - \frac{r^4}{4R}\right)$

$= \frac{3Q}{4R^3}\left(\frac{1}{3}r^3 - \frac{r^4}{4R}\right)$

$q = \frac{3Qr^3}{4R^3}\left(\frac{1}{3} - \frac{r}{4R}\right)$

$E(4\pi r^2) = \frac{3Qr^3}{4R^3}\left(\frac{1}{3} - \frac{r}{4R}\right)$

$E(r) = \frac{3Qr}{16\pi R^3}\left(\frac{1}{3} - \frac{r}{4R}\right)$

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