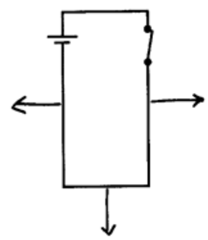
Question 3

15 points total | **Distribution of points**

(a) 2 points



For indicating at least one of the magnetic forces shown, perpendicular to any of the three lower arms and in the plane of the loop, and no forces on the upper arm — 1 point

For correctly indicating all three forces shown — 1 point

(b) 3 points

For recognizing the equilibrium condition with the upward spring force balancing the downward magnetic force and NO inclusion of the gravitational force on the loop — 1 point

$F_S = F_M$ or $kx = I\ell B$

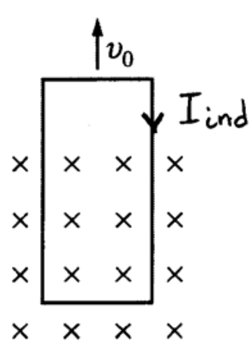
For explicitly solving for the initial magnetic field strength B_0, using the correct expressions for F_S and F_M — 1 point

$$B_0 = \frac{kx}{I\ell}$$

For substituting the width w of the loop bottom for the length ℓ in the magnetic force — 1 point

$$B_0 = \frac{kx}{Iw}$$

(c)
(i) 2 points



For indicating the clockwise induced current direction in the loop — 2 points

Question 3 (continued)

Distribution of points

(c) (continued)

(ii) 3 points

For noting that the potential difference around the loop results from the changing magnetic flux through the loop area — 1 point

$$\varepsilon = -\frac{d\phi_m}{dt} = -B_0\frac{dA}{dt} = -B_0\ell\frac{dy}{dt} = -B_0\ell\upsilon_0$$

For explicitly and correctly solving for the current using Ohm's law and the induced emf — 1 point

$$I_{ind} = \frac{\varepsilon}{R} \quad \text{or} \quad I_{ind} = \frac{d\phi_m}{dt}\Big/R \quad \text{or equivalent}$$

For identifying the length ℓ in the motional emf relationship as the width w of the loop bottom — 1 point

$$I_{ind} = \frac{B_0 w\upsilon_0}{R}$$

(d) 2 points

For recognizing that the current from part (c) (ii) is the appropriate current — 1 point

For using the current from part (c) (ii) in an algebraically correct expression for power — 1 point

$$P = I_{ind}V = I_{ind}^2R = \frac{V^2}{R}$$

$$P = \left(\frac{B_0 w\upsilon_0}{R}\right)^2 R = \frac{B_0^2 w^2\upsilon_0^2}{R}$$

Alternate solution: — *Alternate points*

The external force must balance the force on the current carrying wire in a magnetic field.

$$F_{\text{ext}} = F_B = B_0 w I_{ind}$$

For substituting the current from (c) (ii) into the expression for force — *1 point*

$$F_{ext} = F_B = B_0 w\left(\frac{B_0 w\upsilon_0}{R}\right) = \frac{B_0^2 w^2\upsilon_0}{R}$$

For using the expression for power in terms of constant force and constant speed and substituting the correct expressions for force and speed. — *1 point*

$$P = F\upsilon$$

$$P = \left(\frac{B_0^2 w^2\upsilon_0}{R}\right)\upsilon_0 = \frac{B_0^2 w^2\upsilon_0^2}{R}$$

Question 3 (continued)

Distribution of points

(e) 3 points

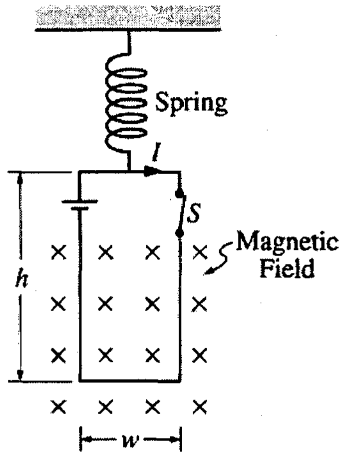
For choosing "Increases" — 1 point

For a clear and complete justification indicating that the magnetic field affects the force directly through the relationship $F_M = B_0 I_{ind} \ell$ and also through the induced current $I_{ind} = B_0 w v_0 / R$ — 2 points

Example:

An increased magnetic field causes the magnetic force on the loop bottom to be larger for a given current and wire length. The larger field also increases the size of the induced current. Thus, the motion must be balanced by a larger applied force to keep the loop moving at the same constant speed v_0 through a larger field with a larger induced current.

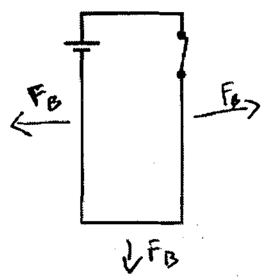
Note: *A partially complete or unclear justification was awarded 1 point.*



E&M 3.

A loop of wire of width w and height h contains a switch and a battery and is connected to a spring of force constant k, as shown above. The loop carries a current I in a clockwise direction, and its bottom is in a constant, uniform magnetic field directed into the plane of the page.

(a) On the diagram of the loop below, indicate the directions of the magnetic forces, if any, that act on each side of the loop.



(b) The switch S is opened, and the loop eventually comes to rest at a new equilibrium position that is a distance x from its former position. Derive an expression for the magnitude B_0 of the uniform magnetic field in terms of the given quantities and fundamental constants.

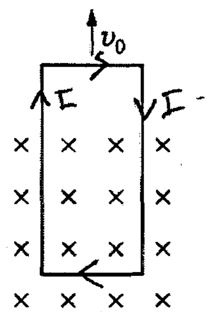
$I w B_0 = kx$

$\boxed{B_0 = \frac{kx}{Iw}}$

The spring and loop are replaced with a loop of the same dimensions and resistance R but without the battery and switch. The new loop is pulled upward, out of the magnetic field, at constant speed v_0. Express algebraic answers to the following questions in terms of B_0, v_0, R, and the dimensions of the loop.

(c)

i. On the diagram of the new loop below, indicate the direction of the induced current in the loop as the loop moves upward.



ii. Derive an expression for the magnitude of this current.

$$\mathcal{E} = \frac{d\Phi}{dt} = B_0 \frac{dA}{dt} = B_0 \frac{dh}{dt} w = B_0 v_0 W$$

$$\mathcal{E} = Ir \rightarrow I = \frac{\mathcal{E}}{R} = \boxed{\frac{B_0 v_0 W}{R}}$$

(d) Derive an expression for the power dissipated in the loop as the loop is pulled at constant speed out of the field.

$$P = I^2 R = \left(\frac{B_0^2 v_0^2 w^2}{R^{\cancel{2}}}\right)\cancel{R} = \boxed{\frac{B_0^2 v_0^2 w^2}{R}}$$

(e) Suppose the magnitude of the magnetic field is increased. Does the external force required to pull the loop at speed v_0 increase, decrease, or remain the same?

_X_ Increases _____ Decreases _____ Remains the same

Justify your answer.

The Force required to pull the Loop at a constant speed would be equal to the Force generated by the magnetic field which pulls down on the loop. If B increases, then the induced current, $\frac{B_0 v_0 w}{R}$, does so as well. Since the downward Force is proportional to both this and the strength of the Field ($F = ILB = \frac{B_0^2 v_0 w L}{R}$), the Force required to contract it is larger

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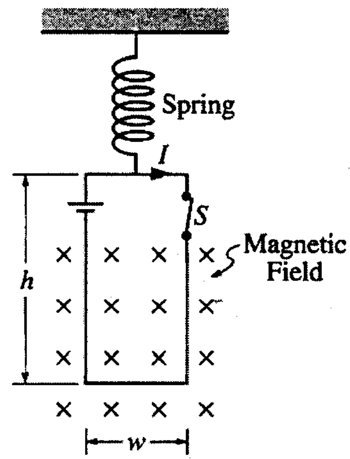
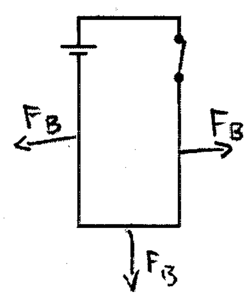


E&M 3.

A loop of wire of width w and height h contains a switch and a battery and is connected to a spring of force constant k, as shown above. The loop carries a current I in a clockwise direction, and its bottom is in a constant, uniform magnetic field directed into the plane of the page.

(a) On the diagram of the loop below, indicate the directions of the magnetic forces, if any, that act on each side of the loop.



(b) The switch S is opened, and the loop eventually comes to rest at a new equilibrium position that is a distance x from its former position. Derive an expression for the magnitude B_0 of the uniform magnetic field in terms of the given quantities and fundamental constants.

$$F_S = F_M$$

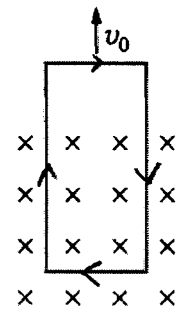
$$-kx = B_0 I h$$

$$B_0 = \frac{-kx}{Ih}$$

The spring and loop are replaced with a loop of the same dimensions and resistance R but without the battery and switch. The new loop is pulled upward, out of the magnetic field, at constant speed v_0. Express algebraic answers to the following questions in terms of B_0, v_0, R, and the dimensions of the loop.

(c)

i. On the diagram of the new loop below, indicate the direction of the induced current in the loop as the loop moves upward.



ii. Derive an expression for the magnitude of this current.

$V = IR$ $\quad I = BLV/r$

$I = B_0 h v_0 / R$

(d) Derive an expression for the power dissipated in the loop as the loop is pulled at constant speed out of the field.

$P = IV$

$P = \frac{B_0 h v_0}{R} \cdot (B_0 h v_0)$

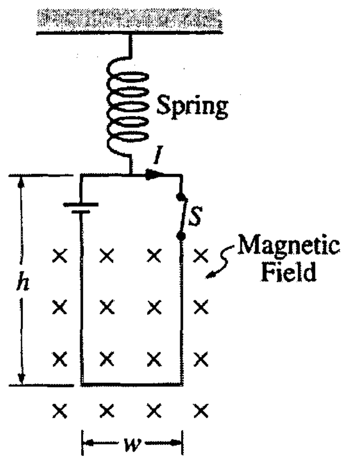
$P = \frac{B_0^2 h^2 v_0^2}{R}$

(e) Suppose the magnitude of the magnetic field is increased. Does the external force required to pull the loop at speed v_0 increase, decrease, or remain the same?

____ Increases __✓__ Decreases ____ Remains the same

Justify your answer.

$F_M = q v \times B$



E&M 3.

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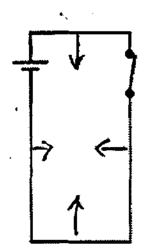


Lenz's law

(b) The switch S is opened, and the loop eventually comes to rest at a new equilibrium position that is a distance x from its former position. Derive an expression for the magnitude B_0 of the uniform magnetic field in terms of the given quantities and fundamental constants.

$\oint B \cdot dl = \mu_0 I_{enc}$ $\quad dl = dh$

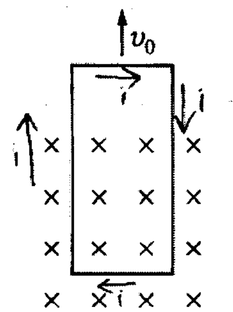
$\frac{B_0 h^2}{2} = \mu_0 I_{enc}$

$B_0 = \frac{2\mu_0 I}{h^2}$

The spring and loop are replaced with a loop of the same dimensions and resistance R but without the battery and switch. The new loop is pulled upward, out of the magnetic field, at constant speed v_0. Express algebraic answers to the following questions in terms of B_0, v_0, R, and the dimensions of the loop.

(c)

i. On the diagram of the new loop below, indicate the direction of the induced current in the loop as the loop moves upward.



ii. Derive an expression for the magnitude of this current.

$$\oint B \cdot dl = \mu_0 I \qquad dl = dh$$

$$\frac{B_0 h^2}{2} = \mu_0 I$$

$$I = \frac{B_0 h^2}{2\mu_0}$$

(d) Derive an expression for the power dissipated in the loop as the loop is pulled at constant speed out of the field.

$$P = IV = I^2 R \qquad P = \left(\frac{B_0 h^2}{2\mu_0}\right)^2 R \cdot V_0$$

$$P = FV$$

$$P = \frac{B_0^2 h^4 R V_0}{4\mu_0^2}$$

(e) Suppose the magnitude of the magnetic field is increased. Does the external force required to pull the loop at speed v_0 increase, decrease, or remain the same?

__✓__ Increases _____ Decreases _____ Remains the same

Justify your answer.

Question 3

Overview

This question probed student understanding of magnetic forces and induced emf. To correctly solve parts (a) and (b), students had to understand how to determine the magnitude and direction of the magnetic forces acting on current carrying wires. To correctly solve parts (c), (d), and (e), students had to work with the induced emf in a loop moving through a magnetic field, calculating the generated current and power dissipated, as well as justifying the magnetic-field dependence of the external force required to move the loop at constant speed.

Sample: E3A
Score: 15

This very clearly organized response includes an excellent justification in part (e) and received full credit on all the parts.

Sample: E3B
Score: 10

Part (a) received full credit. Part (b) earned the points for recognizing that the downward magnetic force would be balanced by the upward spring force and for solving for the magnetic field using correct expressions for the forces, but the final point was lost by incorrectly substituting the height h, instead of the width w, for the length ℓ in the expression for the magnetic force. Part (c)(i) is correct, but part (c)(ii) lost a point for substituting the wrong length in the expression for the induced emf. Part (d) received full credit for an answer consistent with the result in part(c)(ii), but no credit was awarded for part (e).

Sample: E3C
Score: 4

No credit was given for part (a). The student takes an incorrect approach using Ampere's law to part (b) and also received no credit. Part (c)(i) is correct and received 2 points, but part (c)(ii) received no credit for another incorrect attempt to apply Ampere's law. Although a correct expression for the power is initially written in part (d), the expression for the power used in the derivation included an extra factor v_0, so the only point awarded in this part was for the correct substitution of the answer to part (c)(ii) for the current. The last point was given in part (e) for indicating that the force "Increases," but without a justification no further credit could be awarded.