



AP[®] Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary Set 2

Inside:

Free-Response Question 3

- Scoring Guidelines**
- Student Samples**
- Scoring Commentary**

Question 3: Free-Response Question**15 points**

- (a) For a loop rule expression that includes terms for the equivalent resistance $2R$ and the potential difference across the battery **1 point**
-
- For an expression that includes charge Q in the term relating the potential difference across the capacitor and includes charge per unit time $\frac{dQ}{dt}$ in the term relating the potential difference across the pair of resistors **1 point**

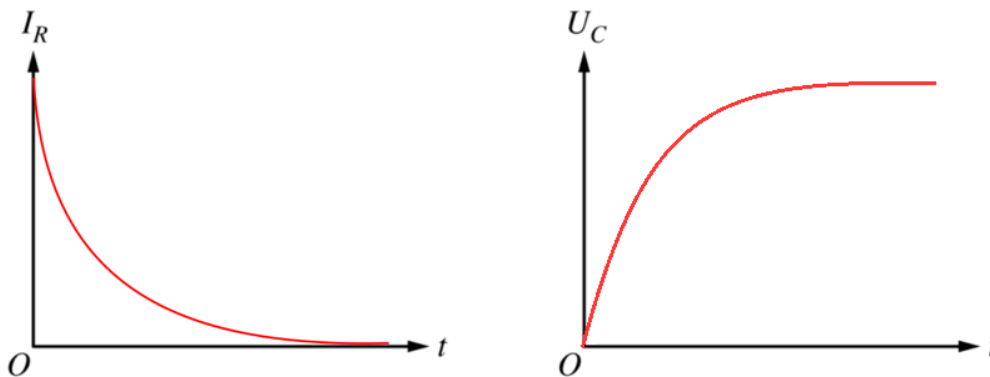
Example Response

$$\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0$$

$$\mathcal{E} - \frac{Q}{C} - 2R \frac{dQ}{dt} = 0$$

Total for part (a) 2 points

- (b) For sketching a concave up and decreasing curve on the graph of I_R as a function of t **1 point**
-
- For sketching a concave down and increasing curve on the graph of U_C as a function of t **1 point**
-
- For sketching both curves that approach a slope of zero as time increases **1 point**

Scoring Note: The third point can be earned even if the first two points are not earned.**Example Response****Total for part (b) 3 points**

-
- (c)(i) For a correct justification that could include one of the following: **1 point**
- An indication that the current is upwards that includes a statement that indicates that positive charge had accumulated on the top plate of Capacitor 1 and/or negative charge has accumulated on the bottom plate of Capacitor 1 when the switch was closed to Position A
 - An indication that the current is upwards that includes a statement that indicates that the value of the electric potential of the top plate of Capacitor 1 is larger than the electric potential of the bottom plate of Capacitor 1 when the switch was closed to Position A
-

Example Responses

When the switch is closed at time t_1 , positive charge has built up on the top plate of the capacitor. This positive charge on the top plate pushes charge up through Resistor 1 and down through Capacitor 2 to charge Capacitor 2.

OR

After a long-time the top plate of Capacitor 1 is at a high potential due to its being charged by the battery. When the switch is closed at time t_1 , the resulting current is up through Resistor 1 as the current goes from high potential on the top plate to low potential clockwise around the circuit through Capacitor 2.

- (c)(ii) For indicating that the total charge on the positive plate of Capacitor 2 is $\frac{Q_0}{2}$ **1 point**
-

Scoring Note: This point can be earned without supporting calculations.

Example Response

The potential difference across Capacitor 1 is equal to the potential difference across Capacitor 2. Capacitor 2 has the same capacitance of Capacitor 1. Therefore, Capacitor 2 stores the same charge that is stored on Capacitor 1. Due to conservation of charge, Capacitor 2 stores half of the original charge equal to $\frac{Q_0}{2}$.

- (c)(iii)** For indicating that the total energy dissipated by Resistor 1 is the difference between the initial electric potential energy of the system at time $t = t_1$ and the final electric potential energy of the system after the new steady state conditions have been reached **1 point**

Example Response

$$\Delta E_R = U_C - U_{0C}$$

- For indicating that only Capacitor 1 stores nonzero electric potential energy initially and both capacitors store electric potential energy after the new steady-state conditions have been reached, or an alternate response that is consistent with part (c)(ii) **1 point**

Example Response

$$U_{0C} = U_{01} \text{ AND } U_C = U_1 + U_2$$

- For correct substitutions for the charges stored on the capacitors after the new steady state conditions have been reached based on part (c)(ii) **1 point**

Example Solution

$$\Delta E_R = U_C - U_{0C}$$

$$\Delta E_R = \left(\frac{1}{2} \left(\frac{1}{C} \right) \left(\frac{Q_0}{2} \right)^2 + \frac{1}{2} \left(\frac{1}{C} \right) \left(\frac{Q_0}{2} \right)^2 \right) - \frac{1}{2} \frac{Q_0^2}{C}$$

$$\Delta E_R = \frac{Q_0^2}{4C} - \frac{1}{2} \frac{Q_0^2}{C}$$

$$\Delta E_R = -\frac{Q_0^2}{4C}$$

Total for part (c) 5 points

- (d)** For the correct expression for charge on the top plate of Capacitor 2 $\left(Q_2 = \frac{2Q_0}{3} \right)$ **1 point**

Example Response

$$C_1 = C \text{ and } C_2 = \kappa C = 2C$$

$$\Delta V_{C1} = \Delta V_{C2}$$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_1 = \frac{1}{2} Q_2$$

$$Q_1 + Q_2 = Q_0$$

$$\frac{1}{2} Q_2 + Q_2 = Q_0$$

$$Q_2 = \frac{2}{3} Q_0$$

Scoring Note: This point can be earned without supporting calculations.**Total for part (d) 1 point**

-
- (e)(i) For a loop rule equation that includes terms for the potential difference across Resistor 2 and the potential difference across Capacitor 2 with the dielectric **1 point**
-

Example Response

$$\Delta V_{R,2} - \Delta V_{C,2} = 0$$

$$\Delta V_{R,2} = \Delta V_{C,2}$$

For correct substitution of IR for the potential difference across Resistor 2 **1 point**

Example Response

$$\Delta V_{R,2} = IR$$

For correct substitutions of $2C$ for the new capacitance of Capacitor 2 with the dielectric inserted and of the charge consistent with part (d) into the expression for the potential difference across the capacitor **1 point**

Example Response

$$\Delta V_{C,2} = \frac{Q_2}{C_2}$$

$$\Delta V_{C,2} = \left(\frac{2Q_0}{3}\right)\left(\frac{1}{2C}\right)$$

Example Solution

$$\Delta V_{R,2} - \Delta V_{C,2} = 0$$

$$\Delta V_{R,2} = \Delta V_{C,2}$$

$$IR = \left(\frac{2Q_0}{3}\right)\left(\frac{1}{2C}\right)$$

$$IR = \frac{Q_0}{3C}$$

$$I = \frac{Q_0}{3RC}$$

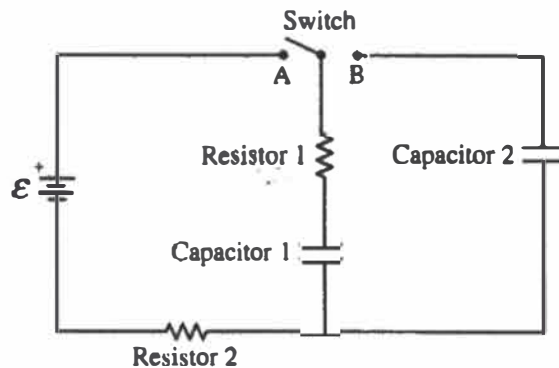
- (e)(ii) For indicating that the current is zero **1 point**
-

Total for part (e) 4 points

Total for question 3 15 points

Question 3

Begin your response to QUESTION 3 on this page.



3. The circuit shown consists of an ideal battery of emf \mathcal{E} , resistors 1 and 2 each with resistance R , capacitors 1 and 2 each with capacitance C , and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

- (a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of \mathcal{E} , R , C , Q , t , and fundamental constants, as appropriate.

$$\mathcal{E} - 2iR - V_{C_1} = 0 \quad V = \frac{Q}{C} \quad I = \frac{dq}{dt} = \frac{dQ}{dt}$$

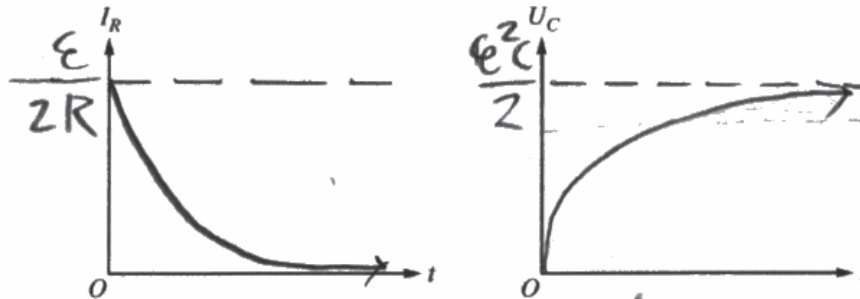
$$\mathcal{E} - 2iR - \frac{Q}{C} = 0$$

$$\mathcal{E} - 2 \frac{dQ}{dt} R - \frac{Q}{C} = 0$$

Question 3

Continue your response to QUESTION 3 on this page.

- (b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

- (c) At time t_1 , the switch is closed to Position B.

- i. Immediately after t_1 , is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

It is up because positive charges accumulated on the top plate of capacitor 1 flow towards (or actually negative charge carriers move the opposite direction) the top plate of capacitor 2.

- ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1 . Express your answer in terms of Q_0 and fundamental constants, as appropriate.

$$Q_0 = q_1 + q_2$$

$$V_1 = V_2$$

$$\frac{q_1}{C} = \frac{q_2}{C} \quad q_1 = q_2$$

$$q_1 = q_2 \text{ so } q_1 + q_2 = Q_0$$

$$\text{so } \boxed{q_2 = \frac{Q_0}{2}}$$

Question 3

Continue your response to QUESTION 3 on this page.

iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C , Q_0 , and fundamental constants, as appropriate.

$$\begin{aligned} \text{Energy dissipated} &= E_0 - E_F \\ E_0 = U_{C0} &= \frac{1}{2} \frac{Q_0^2}{C} \\ E_F = U_{CF} &= 2 \left(\frac{1}{2} \left(\frac{Q_0}{2} \right)^2 / C \right) = \frac{Q_0^2}{4C} \\ E_0 - E_F &= \frac{Q_0^2}{2C} - \frac{Q_0^2}{4C} = \boxed{\frac{Q_0^2}{4C} = E_{\text{dissipated}}} \end{aligned}$$

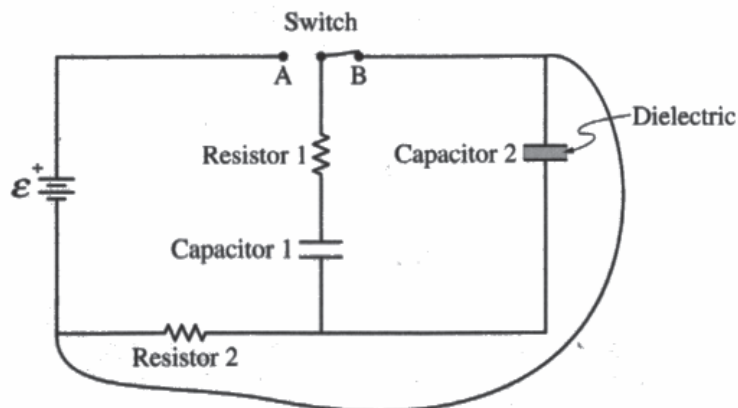
With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

(d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

$$\begin{aligned} C &= \kappa \epsilon_0 A / d \\ \kappa &= 2 \\ \text{so } C_2 &= 2C \\ V_1 &= V_2 \\ Q_1 + Q_2 &= Q_0 \\ \frac{Q_1}{C} &= \frac{Q_2}{2C} \quad \frac{Q_1}{4} = \frac{Q_2}{2C} \quad Q_1 = \frac{Q_2}{2} \\ Q_1 + Q_2 &= Q_0 \\ \frac{Q_2}{2} + Q_2 &= Q_0 \\ \frac{3Q_2}{2} &= Q_0 \\ Q_2 &= \boxed{\frac{2Q_0}{3}} \end{aligned}$$

Question 3

Continue your response to QUESTION 3 on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R , C , Q_0 , and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

$$V_2 = \frac{Q_2}{C_2} = \frac{2 \frac{Q_0}{3}}{2C} = \frac{Q_0}{3C}$$

Loop Rule! $V_2 - \frac{1}{2} \cdot R \cdot I = 0$

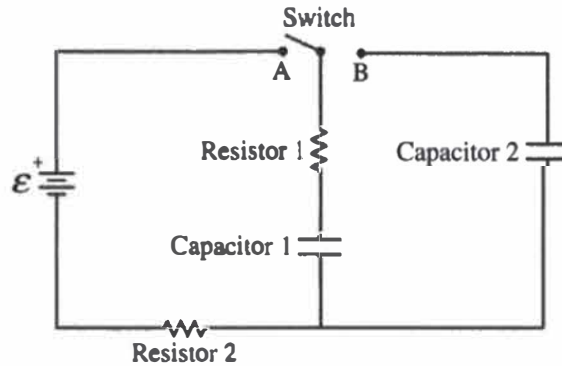
$$I = \frac{V}{R} \quad I_{R_2} = \frac{Q_0}{3C} \div R = \boxed{\frac{Q_0}{3CR}}$$

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

$I = 0$ because the circuit will have reached equilibrium after the capacitor finished discharging

Question 3

Begin your response to QUESTION 3 on this page.



3. The circuit shown consists of an ideal battery of emf \mathcal{E} , resistors 1 and 2 each with resistance R , capacitors 1 and 2 each with capacitance C , and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

- (a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of \mathcal{E} , R , C , Q , t , and fundamental constants, as appropriate.

$$\mathcal{E} - IR_1 - IR_2 - \frac{Q}{C} = 0.$$

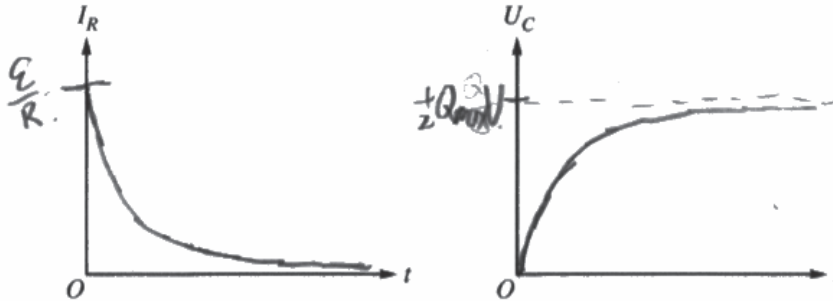
$$\mathcal{E} - 2R \frac{dQ}{dt} - \frac{Q}{C} = 0.$$



Question 3

Continue your response to **QUESTION 3** on this page.

- (b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

- (c) At time t_1 , the switch is closed to Position B.

- i. Immediately after t_1 , is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

Directly up, because when the circuit disconnect with the battery, the current will go clockwise from the positive side of capacitor 1 to the negative side.

- ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1 . Express your answer in terms of Q_0 and fundamental constants, as appropriate.

$$-\frac{Q_0}{C} - IR - \frac{Q}{C} = 0 \quad -\frac{t}{RC} = \ln(Q_0 - Q) \Big|_Q^0$$

$$-\frac{Q_0 - Q}{C} = \frac{dQ}{dt} R \quad Q = Q_0 e^{-\frac{t}{RC}}$$

$$\left(-\frac{1}{RC} dt \right) \int \frac{dQ}{Q_0 - Q}$$

Question 3

Continue your response to **QUESTION 3** on this page.

- iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C , Q_0 , and fundamental constants, as appropriate.

$$U = \frac{1}{2} Q \Delta V$$

$$U = Pt$$

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

- (d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

$$Q = \Delta V C$$

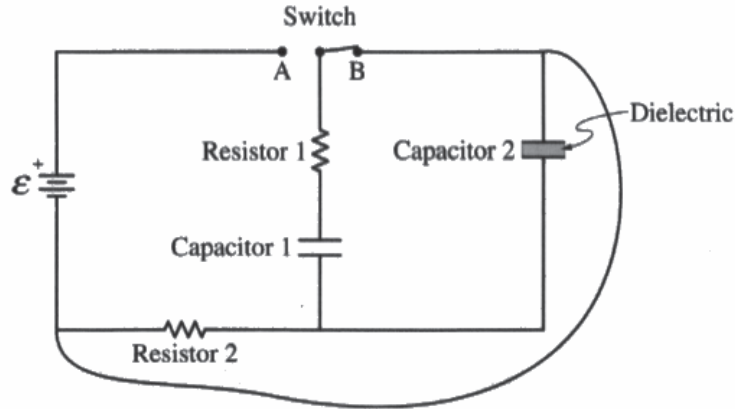
$$Q = \frac{Q_0}{2} \cdot 2C$$

$$Q = \frac{Q_0}{2}$$



Question 3

Continue your response to QUESTION 3 on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R , C , Q_0 , and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

$$-\frac{Q}{C_2} - IR = 0$$

$$IR = -\frac{Q}{C}$$

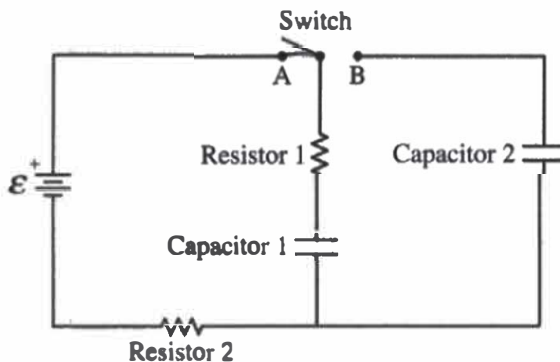
$$I = \frac{Q_0}{RC}$$

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

the current is zero.

Question 3

Begin your response to QUESTION 3 on this page.



3. The circuit shown consists of an ideal battery of emf \mathcal{E} , resistors 1 and 2 each with resistance R , capacitors 1 and 2 each with capacitance C , and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

- (a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of \mathcal{E} , R , C , Q , t , and fundamental constants, as appropriate.

$$I = \frac{dQ}{dt} \quad \mathcal{E} = L \frac{dI}{dt} \quad \mathcal{E} = 2R + C$$

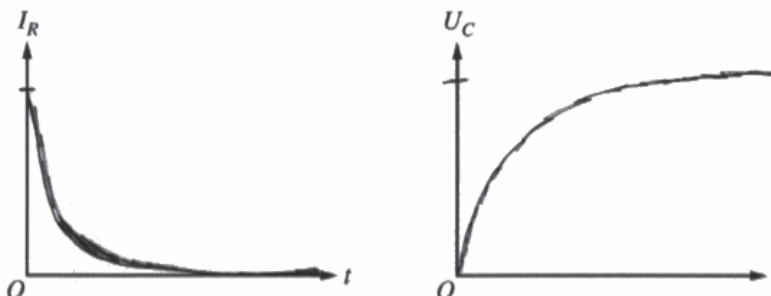
$$\Delta V = \frac{Q}{C} \quad I = \frac{\Delta V}{R} \quad I = \frac{\mathcal{E}}{R}$$



Question 3

Continue your response to **QUESTION 3** on this page.

- (b) On the axes shown, sketch graphs of the current I_R in Resistor 1 and the energy U_C stored in Capacitor 1 as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the total charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

- (c) At time t_1 , the switch is closed to Position B.

- i. Immediately after t_1 , is the direction of the current in Resistor 1 directed up, directed down, or is there no current? Briefly justify your answer.

The direction of the current in resistor resistor 1 is now downward as ~~the~~ capacitor 2 is working to be charged so the current goes up towards capacitor 2 and down resistor 1.

- ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1 . Express your answer in terms of Q_0 and fundamental constants, as appropriate.

$$Q = Q_0$$

Question 3

Continue your response to **QUESTION 3** on this page.

- iii. Derive an expression for the total energy dissipated by Resistor 1 immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C , Q_0 , and fundamental constants, as appropriate.

$$P = I \Delta V \quad \Delta V = \frac{Q}{C}$$

$$U_c = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

$$U_c = \frac{1}{2} \frac{Q_0^2}{C}$$

With the switch still closed to Position B, a dielectric material with dielectric constant $\kappa = 2$ is inserted between the plates of Capacitor 2.

- (d) Determine the charge on the positive plate of Capacitor 2 a long time after the dielectric has been inserted. Express your answer in terms of Q_0 and fundamental constants, as appropriate.

$$C = \frac{\kappa \epsilon_0 A}{d}$$

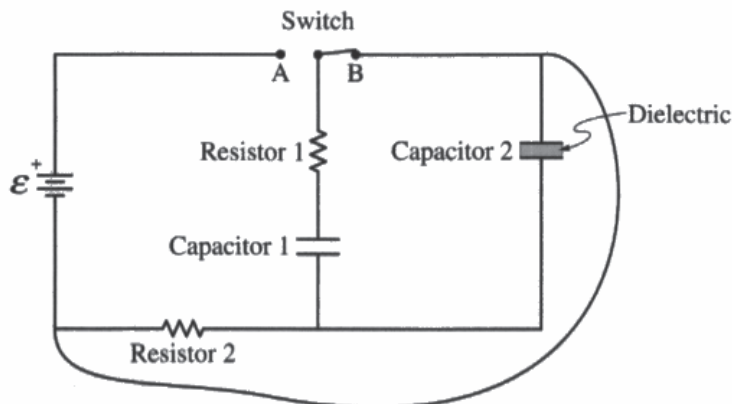
$$\Delta V = \frac{Q}{C}$$

$$Q = 2Q_0$$

$$Q = 2Q_0$$

Question 3

Continue your response to **QUESTION 3** on this page.



With the switch still closed to Position B, a wire of negligible resistance is connected between two corners of the circuit, as shown.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R , C , Q_0 , and fundamental constants, as appropriate.

i. Derive an expression for the current in Resistor 2 immediately after the wire is connected to the circuit.

$$E = \dots$$

$$I = Q_0 e^{-t/\tau}$$

ii. Determine the current in Resistor 2 a long time after the wire is connected to the circuit.

current in resistor 2 a long time after is 0

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Apply Kirchhoff’s Junction Rule (conservation of charge) in a complex RC circuit.
- Apply Kirchhoff’s Loop Rule (conservation of energy) in a complex RC circuit.
- Use Ohm’s law and the definition of capacitance for individual circuit elements.
- Graphically represent the time dependence of quantities in an RC circuit.
- Predict short-term and long-term behavior of an RC circuit while capacitors are charging and discharging.
- Calculate how changing the physical attributes of a capacitor (plate separation distance, presence of a dielectric) affects the stored charge and electric potential energy of a capacitor.
- Skills required in various parts of the response included:
 - Analytical algebraic derivations
 - Proportional reasoning
 - Verbal/prose justification with reasoning
 - Graphical trend display
 - Initial transient circuit behavior
 - Long time duration steady-state circuit behaviors

Sample: 3A

Score: 15

Part (a) earned 2 points. The first point was earned for correctly using the loop rule to write an expression that includes terms for the equivalent resistance, $2R$ and the battery, E . The second point was earned for substituting $\frac{Q}{C}$ for the potential difference across the capacitor and $\frac{dQ}{dt}$ in the term for the potential difference across the pair of resistors. Part (b) earned 3 points. The first point was earned for drawing a curve for the graph of I_R as a function of t is concave up and decreasing. The second point was earned for drawing a curve for the graph of U_C as a function of t is concave down and increasing. The third point was earned for the slopes of both curves approaching zero as time increases. Part (c) earned 5 points. The first point was earned for correctly indicating that positive charge had accumulated on the top plate of Capacitor 1. The second point was earned for correctly indicating that the total charge on the positive plate of Capacitor 2 is $\frac{Q_0}{2}$. The third point was earned for correctly indicating that the energy dissipated is the difference in initial and final energy. The fourth point was earned for correctly indicating that the initial energy is the electric potential energy stored in Capacitor 1, and the final energy is the electric potential energy stored in both capacitors 1 and 2. The fifth point was earned for correctly substituting the correct charges on both capacitors after steady conditions have been reached. Part (d) earned 1 point for correctly determining that the charge on the positive plate of Capacitor 2 is $\frac{2Q_0}{3}$. Part (e) earned 4 points. The first point was earned for correctly using a loop rule that includes the potential difference across Resistor 2 and the potential difference across Capacitor 2. The second point was earned for correctly substituting IR for the potential difference across Resistor 2. The third point was earned for correctly substituting the capacitance $2C$ for Capacitor 2 with the dielectric inserted and charge from part (d) for the potential difference across Capacitor 2. The fourth point was earned for correctly indicating the current in Resistor 2 is zero.

Question 3 (continued)**Sample: 3B****Score: 9**

Part (a) earned 2 points. The first point was earned for correctly using the loop rule to write an expression that includes terms for the equivalent resistance, $2R$ and the battery, E . The second point was earned for substituting $\frac{Q}{C}$ for the potential difference across the capacitor and $\frac{dQ}{dt}$ in the term for the potential difference across the pair of resistors. Part (b) earned 3 points. The first point was earned for drawing a curve for the graph of I_R as a function of t is concave up and decreasing. The second point was earned for drawing a curve for the graph of U_C as a function of t is concave down and increasing. The third point was earned for the slopes of both curves approaching zero as time increases. Part (c) earned 1 point for correctly indicating that positive charge had accumulated on the top plate of Capacitor 1. The second point was not earned because the response incorrectly tries to integrate for the charge and does not indicate that the total charge on the positive plate of Capacitor 2 is $\frac{Q_0}{2}$ under steady-state conditions.

The third point was not earned because the response does not attempt to find a difference in electric potential energy. The fourth point was not earned because, while the response does include a valid electric potential energy equation, it does not indicate that this potential energy is stored in Capacitor 1 initially. The fifth point was not earned because the response does not substitute charge for either capacitor into the electric potential energy expression. Part (d) earned no points because, while the response correctly indicates that the capacitance of Capacitor 2 has doubled, it incorrectly indicates that the potential difference across Capacitor 2 is $\frac{Q_0}{C}$. Part (e) earned 3 points. The first point was earned for correctly using a loop rule that includes the potential difference across Resistor 2 and the potential difference across Capacitor 2. The second point was earned for correctly substituting IR for the potential difference across Resistor 2. The third point was not earned because the response does not correctly substitute the capacitance $2C$ for Capacitor 2 with the dielectric inserted and charge from part (d) for the potential difference across Capacitor 2. The fourth point was earned for correctly indicating the current in Resistor 2 is zero.

Question 3 (continued)**Sample: 3C****Score: 4**

Part (a) earned no points. The first point was not earned because the response does not correctly indicate a valid loop rule to write an expression that includes terms for the equivalent resistance, $2R$ and the battery, E . The second point was not earned because the response does not substitute $\frac{Q}{C}$ for the potential difference across the capacitor and $\frac{dQ}{dt}$ in the term for the potential difference across the pair of resistors. Part (b) earned 3 points. The first point was earned for drawing a curve for the graph of I_R as a function of t is concave up and decreasing. The second point was earned for drawing a curve for the graph of U_C as a function of t is concave down and increasing. The third point was earned for the slopes of both curves approaching zero as time increases. Part (c) earned no points. The first point was not earned because the response incorrectly indicates that negative charge had accumulated on the top plate of Capacitor 1. The second point was not earned because the response does not indicate that the total charge on the positive plate of Capacitor 2 is $\frac{Q_0}{2}$ under steady-state conditions. The third point was not earned because the response does not attempt to find a difference in electric potential energy. The fourth point was not earned because, while the response does include a valid electric potential energy equation, it does not indicate that this potential energy is stored in Capacitor 1 initially. The fifth point was not earned because the response does not substitute charge for either capacitor into the electric potential energy expression. Part (d) earned no points because the response does not correctly indicate the charge on the positive plate of Capacitor 2 is $\frac{2Q_0}{3}$. Part (e) earned 1 point for correctly indicating that the current in Resistor 2 is zero. The second point was not earned because the response does not correctly use a loop rule that includes the potential difference across Resistor 2 and the potential difference across Capacitor 2. The third point was not earned because the response does not substitute IR for the potential difference across Resistor 2. The fourth point was not earned because the response does not correctly substitute the capacitance $2C$ for Capacitor 2 with the dielectric inserted and charge from part (d) for the potential difference across Capacitor 2.