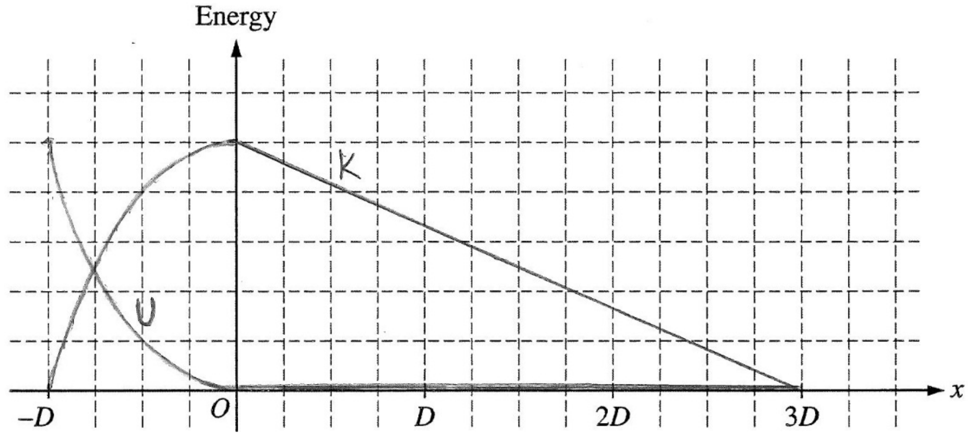
Question 3

12 points total **Distribution of points**

(a) 4 points



For sketching either energy curve with a reasonably correct shape between $x = -D$ and $x = 0$, with zero and maximum values at the correct locations — 1 point

For sketching two curves from $x = -D$ to $x = 0$ with shapes and values such that the total energy is constant (even if the curves are incorrect) — 1 point

For sketching potential energy equal to zero from $x = 0$ to $x = 3D$ — 1 point

For sketching kinetic energy as a linear function from its maximum value at $x = 0$ to zero at $x = 3D$ — 1 point

(b)

(i) 1 point

For identifying that the student is correct that the block will have more energy when it leaves the spring — 1 point

(ii) 1 point

For identifying that the student is incorrect about the new final position of the block because the spring's energy does not scale linearly with its compression — 1 point

Question 3 (continued)

Distribution of points

(c) 3 points

For indicating that the final energy in the spring (which becomes the mechanical energy of the block as it reaches the rough track) is four times the original energy in the spring — 1 point

For indicating that the frictional force remains the same — 1 point

For equating the initial energy in the spring to an expression that shows that the energy dissipated by friction is proportional to the distance the block slides down the rough track — 1 point

Example:

$U_1 = \frac{1}{2}kD^2$ and $U_2 = \frac{1}{2}k(2D)^2$ so $U_2 = 4U_1$

$W_1 = \mu mg(3D)$ and $W_2 = \mu mg\Delta x_2$

$W_1 = U_1$ and $W_2 = U_2 = 4U_1 = 4W_1$

$\mu mg\Delta x_2 = 4(\mu mg(3D))$

$\Delta x_2 = 4(3D) = 12D$

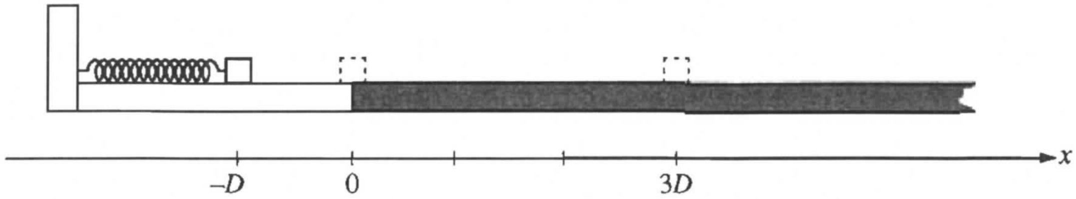
(d) 3 points

For indicating that the student's correct reasoning that the block has more energy in the second situation is expressed by the calculations comparing the initial energy in the spring — 1 point

For indicating that the student's correct reasoning that the block will slide farther is expressed by an equation that indicates that the work done by friction to stop the block in the second situation is some factor greater than the work done in the first situation — 1 point

For indicating that the student's incorrect reasoning that energy scales linearly with the spring's compression is corrected by the expression for the initial energy of the spring — 1 point

P1Q3 A1



3. (12 points, suggested time 25 minutes)

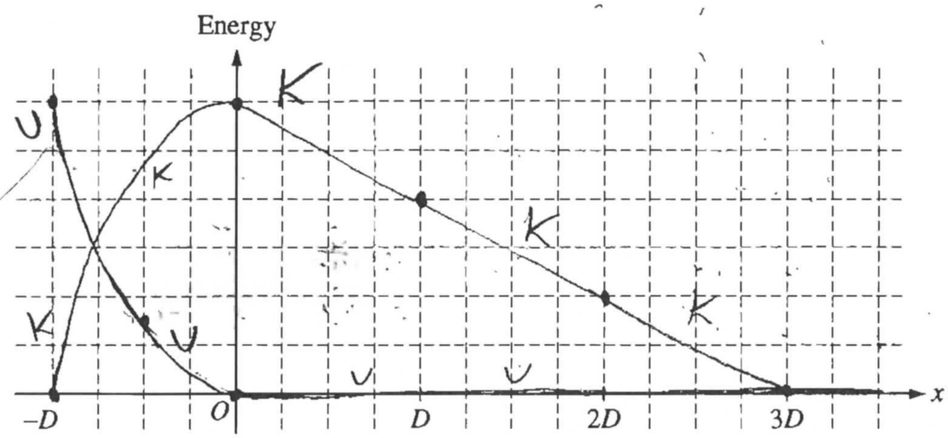
A block is initially at position $x = 0$ and in contact with an uncompressed spring of negligible mass. The block is pushed back along a frictionless surface from position $x = 0$ to $x = -D$, as shown above, compressing the spring by an amount $\Delta x = D$. The block is then released. At $x = 0$ the block enters a rough part of the track and eventually comes to rest at position $x = 3D$. The coefficient of kinetic friction between the block and the rough track is μ.

glasses

(a) On the axes below, sketch and label graphs of the following two quantities as a function of the position of the block between $x = -D$ and $x = 3D$. You do not need to calculate values for the vertical axis, but the same vertical scale should be used for both quantities.

i. The kinetic energy K of the block

ii. The potential energy U of the block-spring system



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The spring is now compressed twice as much, to $\Delta x = 2D$. A student is asked to predict whether the final position of the block will be twice as far at $x = 6D$. The student reasons that since the spring will be compressed twice as much as before, the block will have more energy when it leaves the spring, so it will slide farther along the track before stopping at position $x = 6D$.

(b)

i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

The student is correct to say that compressing the spring will result more a larger kinetic energy value for the block when it leaves the spring. Springs store more energy when compressed further and that greater amount of energy will be transfered to the block.

ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

The student is incorrect to suppose that the block will go twice as far. The amount of energy stored in a spring is proportional to the amount it is compressed squared. Because it is compressed twice as far, it will four times the potential energy which will be transfered to the block, which will also have four times as much energy and will go four times as far.

(c) Use quantitative reasoning, including equations as needed, to develop an expression for the new final position of the block. Express your answer in terms of D. d = final position

$E_K = E_U$ $\quad E_U = F_f \cdot d = mg\mu \cdot d$

$\Sigma F_y = F_N - mg = 0$

$E_k = \frac{1}{2}kx^2$ $\quad \Sigma F_x = F_f = F_N\mu$

$F_f = mg\mu$

$\frac{1}{2}kx^2 = mg\mu d$ $\quad d = \frac{kx^2}{mg\mu}$

$d_1 = \frac{k}{mg\mu} \cdot D^2 = 3D$

$d_2 = \frac{k(2D)^2}{mg\mu} = 4\left(\frac{kD^2}{mg\mu}\right) = 4(3D) = 12D$

Four times the energy will be stored in the spring, four times the energy will go into the block, and friction will take 4 times the distance to disperse the energy.

(d) Explain how any correct aspects of the student's reasoning identified in part (b) are expressed by your mathematical relationships in part (c). Explain how your relationships in part (c) correct any incorrect aspects of the student's reasoning identified in part (b). Refer to the relationships you wrote in part (c), not just the final answer you obtained by manipulating those relationships.

More potential energy will be stored in the spring when it is compressed further as the student stated and as illustrated by the relationship $E_U = \frac{1}{2}k\Delta x^2$. This relationship also shows that when the spring is compressed twice as far (Δx is doubled) four times the energy is stored in the spring. The force of friction is the same in both instances, so the same amount of energy is dissipated by friction per unit of length (demonstrated by $E = F_f \cdot d$) so 4 times the energy will take 4 times the distance to be dissipated, not twice the distance as the student who assumed that potential energy is proportional to Δx supposed.

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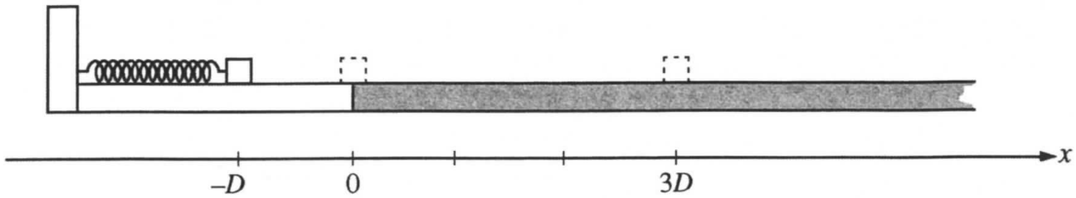


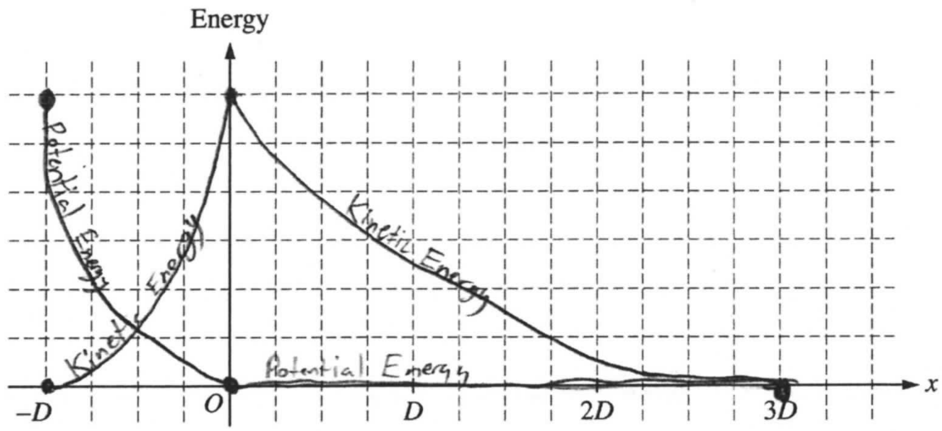
3. (12 points, suggested time 25 minutes)

A block is initially at position $x = 0$ and in contact with an uncompressed spring of negligible mass. The block is pushed back along a frictionless surface from position $x = 0$ to $x = -D$, as shown above, compressing the spring by an amount $\Delta x = D$. The block is then released. At $x = 0$ the block enters a rough part of the track and eventually comes to rest at position $x = 3D$. The coefficient of kinetic friction between the block and the rough track is μ.

(a) On the axes below, sketch and label graphs of the following two quantities as a function of the position of the block between $x = -D$ and $x = 3D$. You do not need to calculate values for the vertical axis, but the same vertical scale should be used for both quantities.

i. The kinetic energy K of the block

ii. The potential energy U of the block-spring system



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The spring is now compressed twice as much, to $\Delta x = 2D$. A student is asked to predict whether the final position of the block will be twice as far at $x = 6D$. The student reasons that since the spring will be compressed twice as much as before, the block will have more energy when it leaves the spring, so it will slide farther along the track before stopping at position $x = 6D$.

(b)

i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

The student is correct that the increased compression results in an increased final position. The increased compression increases spring potential energy. This results in an increased kinetic energy at x=0. This requires the friction force to do more work, and yields a longer stopping distance.

ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

The student's reasoning for why the block goes farther is incorrect. The block goes farther because more work was put into it so more work is required to stop it. Friction is constant, so the distance must be a multiple factor of energy, not velocity.

(c) Use quantitative reasoning, including equations as needed, to develop an expression for the new final position of the block. Express your answer in terms of D.

$U_s = KE$

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$\frac{1}{2}k(2D)^2 = \frac{1}{2}mv^2$

$4k(x^2) = 4mv^2$

4 times the energy.

$W = \Delta E$ $\quad f = \mu \cdot mg$

$\Delta E = F \cdot d$

$4\,KE = \mu \cdot mg \cdot 4(3D)$

4x greater, constant, constant

must be 4 times the distance required originally.

12D is needed to stop the block.

(d) Explain how any correct aspects of the student's reasoning identified in part (b) are expressed by your mathematical relationships in part (c). Explain how your relationships in part (c) correct any incorrect aspects of the student's reasoning identified in part (b). Refer to the relationships you wrote in part (c), not just the final answer you obtained by manipulating those relationships.

The student was correct that the block travels farther because it will go 12 D before coming to rest according to $4\,KE = f \cdot 4(3D)$ (constant, constant, constant) but incorrect on its distance before coming to rest, which is actually 12 D. The student was also correct that the block had more energy from being compressed twice as far, which according to $\Delta E = \frac{1}{2}kx^2$, is 4 times the original energy.

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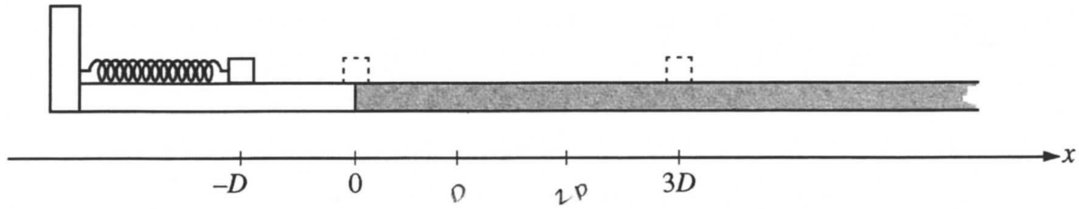

3. (12 points, suggested time 25 minutes)

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$U_s = \frac{1}{2}kx^2$

(a) On the axes below, sketch and label graphs of the following two quantities as a function of the position of the block between $x = -D$ and $x = 3D$. You do not need to calculate values for the vertical axis, but the same vertical scale should be used for both quantities.

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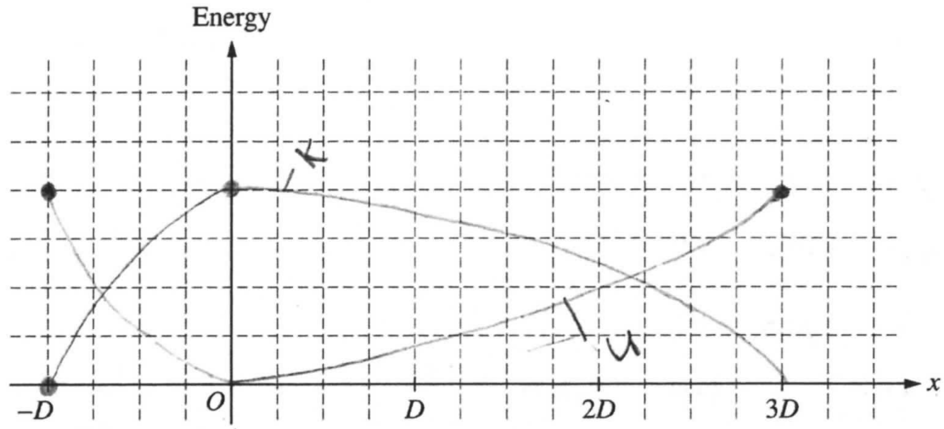

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The spring is now compressed twice as much, to $\Delta x = 2D$. A student is asked to predict whether the final position of the block will be twice as far at $x = 6D$. The student reasons that since the spring will be compressed twice as much as before, the block will have more energy when it leaves the spring, so it will slide farther along the track before stopping at position $x = 6D$.

(b)

i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

The reasoning that the block will move further is correct. The increased Us will allow for the block to have a greater v when it has KE. This increase in velocity will allow it to travel further.

ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

They are incorrect in reasoning that the block will move twice as far. This is because velocity is squared twice when determining ΔX.

(c) Use quantitative reasoning, including equations as needed, to develop an expression for the new final position of the block. Express your answer in terms of D.

$\frac{1}{2}k(2D)^2 = \frac{1}{2}mv^2$ $\frac{1}{2}k4D^2 = \frac{1}{2}mv^2$ V is now 4x larger

$V_f^2 = V_i^2 + 2a\Delta X$

$0 = (4D)^2 + 2a\Delta X$

$\Delta X = 16D$

(d) Explain how any correct aspects of the student's reasoning identified in part (b) are expressed by your mathematical relationships in part (c). Explain how your relationships in part (c) correct any incorrect aspects of the student's reasoning identified in part (b). Refer to the relationships you wrote in part (c), not just the final answer you obtained by manipulating those relationships.

Using part c, we can see that an increase in x did in fact increase the distance traveled b/c it allowed for an increase in velocity, which is used to calculate ΔX. Part c also corrects the student's theory that Δx doubles by proving that it more than doubles because of the kinematics involved.

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Question 3

Overview

The purpose of the question is to examine the relationship between the energy stored in a compressed spring-block system and the work done by the friction force that stops the block once it leaves the spring. The intent is to test the student's understanding of energy principles in multiple representations including graphs, descriptions, and analytical relationships.

Sample: P1Q3 A
Score: 12

The student clearly understands what the question is asking and clearly communicates the reasoning and connections that are asked for by the question. This response received full credit.

Sample: P1Q3 B
Score: 9

Part (a) earned 2 points. The potential energy curve between $-D$ and 0 is correct, but energy is not shown to be conserved in that region. The zero potential energy line is correct, but the kinetic energy line is not straight between 0 and $3D$. Part (b)(i) earned 1 point for full credit. It cites the increased final position as correct, but also refers to increased energy. Part (b)(ii) does not identify the distance as the error in reasoning and earned no credit. Part (c) earned 3 points for full credit. The explanation in part (d) is somewhat indirect, but earned 3 points for full credit.

Sample: P1Q3 C
Score: 5

Part (a) earned 2 points for correct curves between $-D$ and 0. Part (b)(i) earned 1 point for full credit. It cites the increased final position as correct, but also refers to increased energy. Part (b)(ii) has an unclear explanation and earned no credit. Part (c) earned 1 point for calculating the energy in the spring in the new case and recognizing that there is a factor of 4 increase as a result. Part (d) earned 1 point for relating the equations for energy to correct reasoning about energy by reference to part (c) and the increased velocity that is a result of the energy equations.