AP® Calculus AB
2003 Sample Student Responses
Form B

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Work for problem 2(a)

$\int_0^{12} \left(2 + \frac{10}{(1+\ln(t+1))}\right) dt$

$= 70.571$ gallons

Work for problem 2(b)

$H(6) = 2 + \frac{10}{(1+\ln(7))}$

$= 5.395$ gallons coming in

$R(6) = 12\sin\left(\frac{6^2}{47}\right)$

$= 8.319$ gallons being removed

The level of heating oil is falling at $t=6$ hours b/c more gallons are being removed than pumped into the tank. $8.319 > 5.395$ or $H(6) < R(6)$ thus lowering the level of heating oil in the tank.

Continue problem 2 on page 7.

Work for problem 2(c)

$\int_0^{12}\left(2+\frac{10}{(1+\ln(t+1))}\right)dt = 70.571$ gallons pumped in

$\int_0^{12}\left(12\sin\left(\frac{t^2}{47}\right)\right)dt = 73.545$ gallons removed

125 gallons originally

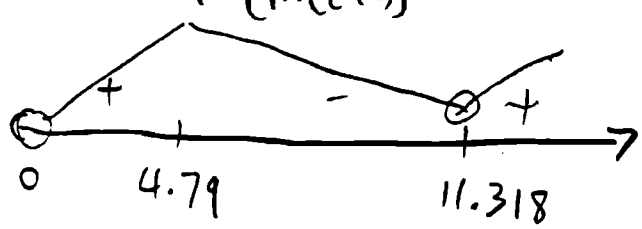
$125 + 70.571 - 73.545 = 122.026$ gallons

Work for problem 2(d)

$V = 125 + \int_0^{12}\left(2+\frac{10}{(1+\ln(t+1))}\right)dt - \int_0^{12}\left(12\sin\left(\frac{t^2}{47}\right)\right)dt$

$\frac{dV}{dt} = 2 + \frac{10}{1+\ln(t+1)} - 12\sin\left(\frac{t^2}{47}\right)$

$0 = 2 + \frac{10}{1+(\ln(t+1))} - 12\sin\left(\frac{t^2}{47}\right)$



$V(11.318) = 125 + 58.118 - 58.207 = 124.77$

$V(0) = 125$

Minimum volume at $t = 11.318$. Two local min at $t = 0$ and $t = 11.318$. $V(0) > V(11.318)$ so $t = 11.318$ is when volume of heating oil the least.

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B,

Work for problem 2(a)

$$H(t) = 2 + \frac{10}{1+\ln(t+1)}$$

$$\int_0^{12} 2 + \frac{10}{1+\ln(t+1)}$$

$$= 70.571$$

70.571 gallons of heating oil are pumped in after 12 hours.

Work for problem 2(b)

Adding oil at $2 + \frac{10}{1+\ln(t+1)}$

extracting oil at $12\sin\left(\frac{t^2}{47}\right)$

Net Oil Δ $= \left[2 + \frac{10}{1+\ln(t+1)}\right] - \left[12\sin\left(\frac{t^2}{47}\right)\right]$

$O(x)$

$O(6) = 8.594$

The level of the tank is rising at $t=6$.
for we have been given the rates of change for the oil in the tank. As long as $O(t)$ is positive oil is being added to the tank.
We have been basically been given the derivative for the amount of oil in the tank.

Continue problem 2 on page 7.

Work for problem 2(c)

At $t=12$ hours there will be:

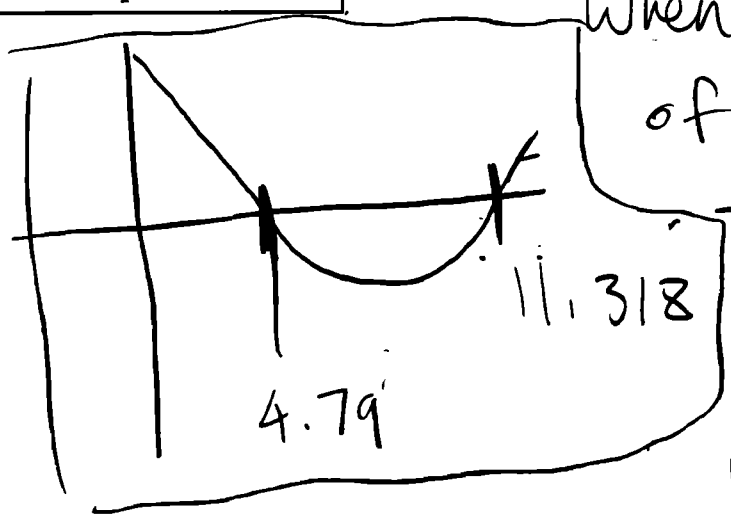
$\int_0^{12} O(t) + 125$ gallons of oil in the tank

$$Vol = \int_0^{12} \left[2 + \frac{10}{1+Ln(t+1)} - \left[12\sin\left(\frac{t^2}{47}\right)\right]\right] + 125$$

$Vol = -2.974 + 125$

Volume in tank after 12 hours = 122.026 gallons

Work for problem 2(d)



When the derivative (Rate of change of the amount of oil in the tank is at O, that is when there is a max and a min in the volume in the tank. If there is a sign change from positive to negative it is a max if it is a change from negative to positive it is a min. Therefore $O(t) = 0$, at two different t values between 0 and 12. 4.79 and 11.318. At 11.318 there is a sign change from negative to positive so that is the location of the minimum point.

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The volume of oil in the tank at $t=11.318$ hours is the least amount of oil in the tank