



AP[®] Calculus AB 2003 Sample Student Responses Form B

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| | | | | | | | |
|----------------------------|----|----|-----|-----|-----|-----|-----|
| Distance x (mm) | 0 | 60 | 120 | 180 | 240 | 300 | 360 |
| Diameter $B(x)$ (mm) | 24 | 30 | 28 | 30 | 26 | 24 | 26 |

Work for problem 3(a)

Since radius = $\frac{1}{2}$ (diameter)

$$\Rightarrow \text{Average radius} = \frac{1}{2} \left(\frac{1}{(360-0)\text{mm}} \int_0^{360} B(x) dx \right) = \frac{1}{720\text{mm}} \int_0^{360} B(x) dx$$

Work for problem 3(b)

$$\int_0^{360} B(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(c_k) \Delta x$$

$$c_1 = 60\text{mm} \Rightarrow f(c_1) = 30\text{mm}$$

$$c_2 = 180\text{mm} \Rightarrow f(c_2) = 30\text{mm}$$

$$c_3 = 300\text{mm} \Rightarrow f(c_3) = 24\text{mm}$$

$$\Delta x = \frac{360\text{mm}}{n} = \frac{360\text{mm}}{3} = 120\text{mm}$$

$$\Rightarrow \sum_{k=1}^3 f(c_k) \Delta x = 120\text{mm} (f(c_1) + f(c_2) + f(c_3))$$

$$= 120\text{mm} (30\text{mm} + 30\text{mm} + 24\text{mm}) = 10080\text{mm}^2$$

$$\Rightarrow \text{Average radius} = \frac{1}{720} \int_0^{360} B(x) dx \approx \frac{1}{720\text{mm}} (10080\text{mm}^2) = 14\text{mm}$$

Continue problem 3 on page 9.

Work for problem 3(c)

It is the volume of blood in the blood vessel starting from a distance of 125mm from 1 end to a distance of 275 mm from the same end. The units will be (mm)³

Work for problem 3(d)

$$B''(x) = 0 \Rightarrow \frac{B'(b) - B'(a)}{b - a} = 0$$

b, a, d, c, e, f ∈ (0, 360)
c > f, d > e, b > a

$$\Rightarrow B'(b) = B'(a)$$

$$\Rightarrow \frac{B(d) - B(e)}{d - e} = \frac{B(c) - B(f)}{c - f}$$

Since for all x, s+x is the same

$$\Rightarrow B(d) - B(e) = B(c) - B(f)$$

From the table there are values of d, e, c, f such that

$$B(d) - B(e) = B(c) - B(f) \text{ . For example at } x = 300$$

$$B(360) - B(300) = B(300) - B(240) \Rightarrow 26 - 24 = 26 - 24 \Rightarrow 0 = 0$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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|----------------------------|----|----|-----|-----|-----|-----|-----|
| Distance x (mm) | 0 | 60 | 120 | 180 | 240 | 300 | 360 |
| Diameter $B(x)$ (mm) | 24 | 30 | 28 | 30 | 26 | 24 | 26 |

$$F_{avg} = \frac{1}{b-a} \int_a^b f(x) dx$$

Work for problem 3(a)

$$\begin{aligned}
 B(x)_{avg} &= \frac{1}{360-0} \int_0^{360} \frac{B(x)}{2} dx \\
 &= \frac{1}{360} \int_0^{360} \frac{B(x)}{2} dx
 \end{aligned}$$

Work for problem 3(b)

$$\frac{360}{3} = 120$$

$$\begin{aligned}
 B(x)_{avg} &= \frac{1}{360} \left[\frac{120 \cdot f(60)}{2} + \frac{120 \cdot f(180)}{2} + \frac{120 \cdot f(300)}{2} \right] \\
 &= \frac{120}{360} [15 + 15 + 12] \\
 &= \frac{12}{36} \times 42 \\
 &= 14 \text{ mm}
 \end{aligned}$$

Continue problem 3 on page 9.

Work for problem 3(c)

$\frac{B(x)}{2}$ = radius of blood vessel

$$\pi \int_{125}^{275} \left(\frac{B(x)}{2}\right)^2 dx$$

Volume of the blood vessel from
 $x = 125 \text{ mm}$ to $x = 275 \text{ mm}$ in $(\text{mm})^3$

Work for problem 3(d)

At x where $B''(x) = 0$

There is an inflection on the graph

The sign of $B'(x)$ changes

$B'(x)$ = the change of diameter

From the table we know that when the diameter
increases $B'(x) > 0$ when diameter decrease $B'(x) < 0$

$B'(x)$ changes signs

$\therefore B''(x) = 0$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON
PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.