AP® Calculus AB
2003 Scoring Commentary
Form B

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Question 1

This problem presented two regions. Region *R* was bounded by a graph and the *x*-axis. Region *S* was bounded by the graph, the line tangent to the graph at one point, and the *x*-axis. Explicit formulas for the function whose graph was shown and for the line were given. The questions involved integration to find an area and a volume, as well as differentiation to find the slope of the tangent line at a given point. Part (a) asked students to show that the line was, in fact, tangent to the graph at $x = 3$. Students needed to verify that the line and the graph intersected at $x = 3$ and that the slope of the line was the slope of the line tangent to the graph at that point. Part (b) asked to find the area of the region *S*. This could be done in several ways. One of the easiest was to find the area under the graph from $x = 3$ to $x = 4$ and then subtract that area from the area of the triangle bounded by the line, the vertical line $x = 3$, and the *x*-axis. Part (c) asked to find the volume of a solid of revolution whose cross sections were discs.

Sample A (Score 9)

The student earned all 9 points.

Sample B (Score 7)

The student earned 7 points: 1 point in part (a), 4 points in part (b), and 2 points in part (c). The student lost 1 point in part (a) since $f'(3)$ was not given. In part (c), the student lost 1 point since the answer was rounded to 2 decimal places, instead of the required 3 places.

Question 2

This presented a "real-life" situation in which heating oil is simultaneously being pumped into as well as removed from a tank. Explicit functions described the rates at which oil entered the tank and the rate at which oil left. The problem explored aspects of the Fundamental Theorem of Calculus. In part (a), students needed to recognize that the total amount of oil pumped into the tank during a 12-hour period could be found by evaluating a definite integral of the rate at which oil was pumped into the tank. Students needed to set up the correct integral and evaluate it. Part (b) asked whether the level of oil was rising or falling at a specified time. Students needed to know how to handle the offsetting rates and correctly interpret their difference. Part (c) asked for the number of gallons in the tank at the end of the 12-hour period. This required students to integrate the difference of the rates, remembering to account for the amount of oil in the tank at the beginning of the period. Part (d) was an optimization problem. Students needed to know that the minimum must occur either at the start or end of the period, or when the offsetting rates are equal. Students needed to be able to calculate the amount of oil in the tank at each of these times or to use the signs of $H(t)-R(t)$ to narrow the field of candidates to $t=0$ and $t=11.318,$ and then correctly identify the smallest value.

Sample A (Score 9)

The student earned all 9 points.

Sample B (Score 7)

The student earned 7 points: 2 points in part (a), 0 points in part (b), 3 points in part (c), and 2 points in part (d). In part (b), the student incorrectly stated that the level of oil in the tank is rising. In part (d), the student did not give a complete analysis and lost a point.

Question 3

This problem presented a function, given in tabular form, which represented the diameter of a circular blood vessel at certain distances from one end. This problem tested the ability to approximate, use, and interpret integrals. It also required knowledge of, and the ability to use, the Mean Value Theorem. Part (a) required that students know how to set up an integral that represented the average value of the function. While the given function represents the diameter, the question asks about the average value of the radius. This was intentional. Part (b) required knowledge of how to set up the midpoint Riemann sum as an approximation to a definite integral. Part (c) presented an integral and tested whether students could recognize that this is π times the definite integral of the square of the radius from distance 125 mm to 275 mm, and thus represents the volume of the circular blood vessel taken over this interval. References to the specific interval and to the units, mm^3, were important. Part (d) could be done in several ways. One approach was for students to recognize that if there are two values of x at which B' takes on the same value, then the Mean Value Theorem guarantees a value of x at which $B''(x) = 0.$ There were several ways of showing that there were two values of x at which B' took on the same value. One was to observe that, by the Mean Value Theorem, there is a value on $[60, 180]$ at which B' is zero and another on $[240, 360]$ at which B' is again zero. Another was to observe that B must have a local maximum between $x = 0$ and $x = 120$ and a local minimum between $x = 60$ and $x = 180,$ and B' is zero at both of these local extrema. A third would be to invoke the Mean Value Theorem to show that there is a value of x between 120 and 180 at which B' is $\frac{2}{60},$ and another value of x between 300 and 360 at which B' is again $\frac{2}{60}.$ However, it was incorrect to assume that the function B attained a local extremum at any of the given points.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample B</u> (Score 7)

The student earned 7 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). In part (d), the student earned 1 point for correctly stating that $B'(x) = 0$ at some point x in the interval $(0, 360).$

Question 4

This problem involved particle motion along a straight line, the x-axis. The velocity at time t was given explicitly. Students needed to use differentiation and integration to answer questions about acceleration and distance traveled, and needed to interpret what the velocity function implies about the motion of the particle. Part (a) required students to find the acceleration by differentiating the velocity function. In part (b), students needed to understand the difference between velocity and speed. One approach to finding the correct answer was to know that when both velocity and acceleration are negative, the speed is increasing. In part (c), students needed to find the times at which the velocity changed sign. In part (d), students could either use the information from part (c) or simply integrate the speed function over the time interval.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample C</u> (Score 7)

The student earned 7 points: 2 points in part (a), 1 point in part (b), 2 points in part (c), and 2 points in part (d). The student lost 1 point in part (d) for an incorrect integrand, making the student ineligible for the answer point.

Question 5

This problem presented the graph of a function f and asked questions about the function g defined as the definite integral of f over the interval from 2 to x. While it was possible to find an explicit algebraic representation of f as a piecewise-linear function, the questions were designed to be more easily answered using the Fundamental Theorem of Calculus and the geometric meaning of the integral. Part (a) required the use of the Fundamental Theorem of Calculus, the recognition of integral as area, and the ability to read the derivative of a linear function from its graph. Part (b) asked for the average rate of change of the function g, which required translating the difference in the values of g into a definite integral of f, and evaluating that definite integral from the graph of the function. Students could use $g(3)$ from part (a) and compute $g(0)$ separately. Part (c) required solving $f(c)=\frac{7}{3}$ and recognizing that there are exactly two solutions. Although this looks like a problem involving the Mean Value Theorem, it actually is not, though it could be used to initiate classroom discussion of the Mean Value Theorem. Part (d) required students to find a point of inflection by reading the graph of the derivative.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample B</u> (Score 7)

The student earned 7 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 2 points in part (d). In part (a), the student did not find $g''(3)$, losing that point. In part (c), the student gave a correct justification, but used an incorrect equation, and therefore lost the answer point.

Question 6

This problem presented a separable differentiable equation with an initial condition. Part (a) required using the product and chain rules and recognizing that $f'(3)$ could be found from the differential equation. Part (b) asked to solve the separable differential equation with the given initial condition.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 3 points in part (a) and 4 points in part (b). In part (b), the student did not solve for y correctly and lost 2 points.