



**AP<sup>®</sup> Calculus AB  
2004 Scoring Commentary  
Form B**

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**Question 1**

This question described a region bounded by a parabolic arc and two straight lines. In part (a), students had to find the area of the region. In part (b), they were asked to find the volume of the solid generated when the region was rotated about a horizontal line lying above the region. In part (c), they were asked to find the volume of the solid generated when the region was rotated about the vertical line that forms the right-hand boundary.

Sample B (Score 9)

The student earned all 9 points. Units were not necessary.

Sample C (Score 7)

The student earned 7 points: all points in parts (a) and (b) and 1 point in part (c). In part (c), the student's integrand was incorrect, thus making the student ineligible for the answer point.

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**Question 2**

Students were given a function that models the rate of change of a mosquito population at time  $t$  and the number of mosquitoes at time  $t = 0$ . In part (a), they were asked to show that the number of mosquitoes was increasing at time  $t = 6$ , which could be done by showing that the rate of change was positive at that time. Part (b) asked about the second derivative of the number of mosquitoes: Was the number of mosquitoes increasing at an increasing rate or increasing at a decreasing rate at time  $t = 6$ ? Students needed to give reasons for their answers in terms of the derivative of the rate of change in the mosquito population. Part (c) asked for the number of mosquitoes at time  $t = 31$ , a question that required using the initial value and a definite integral of the rate of change. Part (d) asked for the maximum number of mosquitoes over the closed interval from  $t = 0$  to  $t = 31$ . Because the rate of change of the number of mosquitoes was positive, then negative, then positive, one way to answer this was to compare the number of mosquitoes at the first time when the rate of change was zero (changing from an increasing number of mosquitoes to a decreasing number) and at  $t = 31$ . A student who used this approach needed to be clear about why it was enough to check the number of mosquitoes at just these two times.

Sample B (Score 9)

The student earned all 9 points.

Sample D (Score 7)

The student earned 7 points: all points in parts (a) and (b), 1 point in part (c), and 3 points in part (d). In part (c), the student lost a point for transposing the answer. In part (d), the student did not include the analysis at  $t = 31$ .

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**Question 3**

This question presented a table of values of the velocity of a test plane (in miles per minute) measured at five-minute intervals from  $t = 0$  to  $t = 40$  minutes. Part (a) asked for a midpoint Riemann sum approximation to the definite integral of the velocity function over this 40-minute time interval. Students had to know that the definite integral represented the total distance flown by the airplane during this time. Part (b) asked for the least number of times at which the acceleration could equal zero during the 40 minutes. It is implicitly assumed that acceleration must be a continuous function. Students had to justify their answers. The easiest way to do this was to use the Mean Value Theorem. Since velocities were the same at times  $t = 0$  and  $t = 15$ , acceleration must be zero somewhere strictly between these times. Similarly, acceleration must be zero between times  $t = 25$  and  $t = 30$ . Part (c) gave a model for the velocity at any time  $t$  in the 40-minute interval and asked for acceleration (with units) at time  $t = 23$ . Part (d) asked for the average velocity, based on the model given in part (c), over the 40-minute interval.

Sample B (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 2 points in part (a), 1 point in part (b), the point in part (c), and all points in part (d). The student earned the first point in part (a) for the values of  $v(5) + v(15) + v(25) + v(35)$  and the second point for the correct answer. The third point was not earned because the time interval of 40 minutes was not mentioned. In part (b), the student did not earn the justification point.

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**Question 4**

Students were given the graph of  $f'$  over the interval from  $x = -1$  to  $x = 5$ , were told that it has horizontal tangent lines at  $x = 1$  and at  $x = 3$ , that  $f$  is twice differentiable, and that  $f(2) = 6$ . They had to use this information to answer questions about the function  $f$ . Part (a) asked for the  $x$ -coordinates of the points of inflection of the graph of  $f$  with a reason. Students needed to know that the graph of  $f$  has a point of inflection where the graph of its derivative changes from increasing to decreasing or decreasing to increasing. Part (b) asked for the  $x$ -coordinates of the absolute minimum and maximum values of  $f$  on the closed interval  $[-1, 5]$ . They had to explain how they used the graph of  $f'$  to answer this question. Part (c) defined  $g(x) = xf(x)$  and asked for the equation of the line tangent to the graph of  $g$  at  $x = 2$ . The value of  $f'(2)$  could be read from the graph.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: all points in parts (a) and (c) and 2 points in part (b). In part (b), the student incorrectly identified  $x = 5$  as the  $x$ -value of the absolute maximum, and thus was ineligible for the justification point.

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**Question 5**

This question presented a separable differential equation. Part (a) asked students to sketch a slope field, identifying the slopes at twelve specified points. It was not necessary to draw the slopes precisely, but it was necessary to distinguish between horizontal, increasing, and decreasing tangent line segments. Part (b) probed student understanding of the fact that a slope field only exhibits a small sample of the possible tangent line segments. Students had to use knowledge of the first derivative to determine all points at which the tangent line segments had negative slope. For part (c), students needed to use separation of variables to solve the differential equation with an initial value.

Sample A (Score 9)

The student earned all 9 points.

Sample D (Score 7)

The student earned 7 points: both points in part (a) and 5 points in part (c). The student did not identify the correct region in part (b). The student did not earn the answer point in part (c) because the value 2 (from earlier work) was not included in the final answer.

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**Question 6**

This question presented a region bounded above by the graph of  $y = x^n$ , below by the  $x$ -axis, and to the right by the line tangent to the graph of  $y = x^n$  at  $(1, 1)$ . The problem was to find the value of  $n > 1$  that maximizes this area. Part (a) asked students to find the total area under the curve, as a function of  $n$ , from  $x = 0$  to  $x = 1$ . Part (b) asked students to show that the area of the triangle formed by the tangent line, the  $x$ -axis, and the line  $x = 1$  was  $\frac{1}{2n}$ . Students could use the fact that the slope of the tangent line is  $n$  to conclude that the base has length  $\frac{1}{n}$ . Because the height was 1, the area was  $\frac{1}{2n}$ . Part (c) asked for the area of the region in question, expressed as a function of  $n$ . This was  $\frac{1}{n+1} - \frac{1}{2n}$ . Students were then asked for the value of  $n > 1$  that maximizes this function. The problem was easiest if the terms were not combined over a common denominator, before differentiating.

Sample B (Score 9)

The student earned all 9 points. In part (c), the student chose the correct  $n$  based on the fact that it produced a positive area, rather than showing that only one solution satisfies the criteria  $n > 1$ .

Sample D (Score 7)

The student earned 7 points: 2 points in part (a), 2 points in part (b), and 3 points in part (c). The student did not earn the third point in part (b) because of incorrect algebra in the last line at the bottom of the first page. In part (c), the second and third points are awarded for correctly setting the numerator of the derivative equal to 0. The student did not earn the fourth point in part (c).