



AP[®] Calculus AB 2004 Free-Response Questions

The materials included in these files are intended for noncommercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program[®]. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. This permission does not apply to any third-party copyrights contained herein. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here.

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 4,500 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT[®], the PSAT/NMSQT[®], and the Advanced Placement Program[®] (AP[®]). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

For further information, visit www.collegeboard.com

Copyright © 2004 College Entrance Examination Board. All rights reserved. College Board, Advanced Placement Program, AP, AP Central, AP Vertical Teams, APCD, Pacesetter, Pre-AP, SAT, Student Search Service, and the acorn logo are registered trademarks of the College Entrance Examination Board. PSAT/NMSQT is a registered trademark jointly owned by the College Entrance Examination Board and the National Merit Scholarship Corporation. Educational Testing Service and ETS are registered trademarks of Educational Testing Service. Other products and services may be trademarks of their respective owners.

For the College Board's online home for AP professionals, visit AP Central at apcentral.collegeboard.com.

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

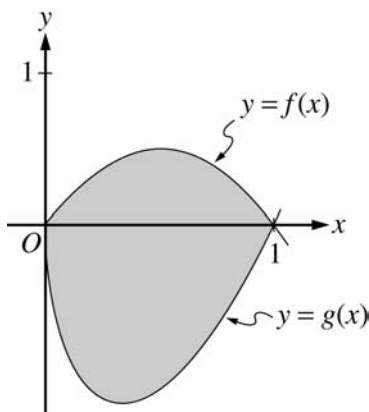
1. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4 \sin\left(\frac{t}{2}\right) \text{ for } 0 \leq t \leq 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

- (a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?
 - (b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.
 - (c) What is the average value of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
 - (d) What is the average rate of change of the traffic flow over the time interval $10 \leq t \leq 15$? Indicate units of measure.
-

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



2. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \leq x \leq 1$. The graphs of f and g are shown in the figure above.
- Find the area of the shaded region enclosed by the graphs of f and g .
 - Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.
 - Let h be the function given by $h(x) = kx(1 - x)$ for $0 \leq x \leq 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x -axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k .
-

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

3. A particle moves along the y -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)
- (a) Find the acceleration of the particle at time $t = 2$.
 - (b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
 - (c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.
 - (d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.
-

END OF PART A OF SECTION II

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

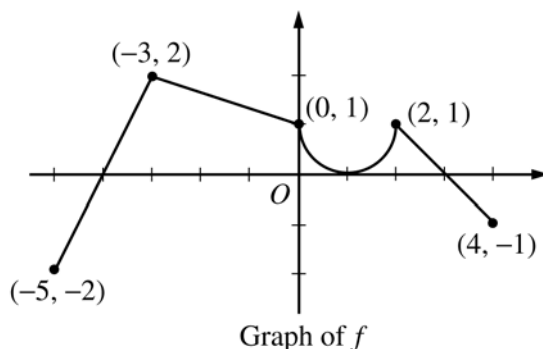
No calculator is allowed for these problems.

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x -coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y -coordinate of P .

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P ? Justify your answer.



5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function

given by $g(x) = \int_{-3}^x f(t) dt$.

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

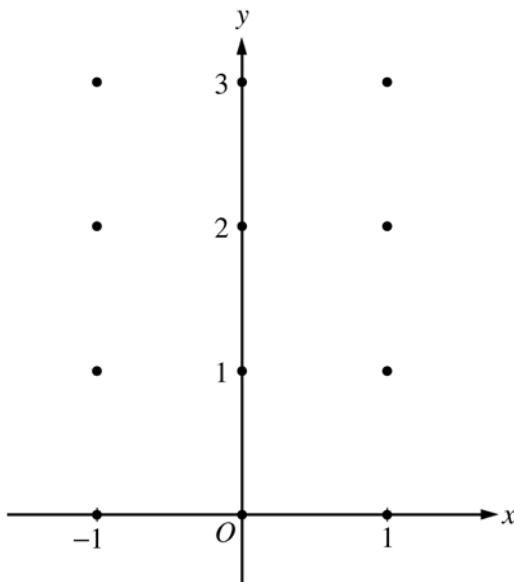
(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

6. Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are positive.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.
-

END OF EXAMINATION