CollegeBoard
Advanced Placement
Program

AP® Calculus AB
2005 Scoring Commentary
Form B

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Question 1

Overview

The student is given a region bounded by the graphs of two curves that intersect at the *y*-axis and at a second point that needs to be determined. The student must find the area of the region in part (a); the volume of the solid generated when the region is rotated about the *x*-axis in part (b); and the volume of the solid above the region for which cross sections perpendicular to the *x*-axis are semicircles in part (c).

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned the limits point and the 2 points in part (a), 1 point in part (b), and 2 points in part (c). Part (a) is correct. In part (b) the student earned one of the integrand points. The answer is neither correct nor consistent with the integral presented. In part (c) the student earned both integrand points but not the answer point.

Sample: 1C
Score: 4

The student earned the limits point and the 2 points in part (a) and 1 point in part (c). Part (a) is correct. In part (b) the student did not earn any points because the integrand is not the difference of squares; thus, the student was not eligible for the answer point. In part (c) the student earned one of the two integrand points.

Question 2

Overview

The problem presents a situation in which water is being pumped into, and removed from, a tank. Functions modeling the rates at which water is entering and leaving the tank are given. Part (a) tests students' ability to combine these rates by asking whether or not the amount of water in the tank is increasing at a specific time and the reason for this. Part (b) tests knowledge of the Fundamental Theorem of Calculus. It requires knowing how to use the rates and the initial amount of water in the tank to determine the amount of water in the tank at a given time. Part (c) asks for the time at which the water is at an absolute minimum. Students can check the water level at the endpoints and at the times when the rates are equal, or they can argue that since the rate of removal is greater than the rate at which water was pumped in over two intervals, one at the beginning and one at the end of the 18-hour period, the minimum can only occur at the first time $t > 0$ at which these curves cross or at time $t = 18$. After making this argument, they can then check the water level at those two times. Part (d) requires setting up an equation in which the integral of the rate of removal from $t = 18$ to $t = k$ is equated to the amount of water the student states is in the tank at time $t = 18$.

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 3 points in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the information from the student-created graph did not earn any points. In part (b) the student correctly figures the number of gallons at time $t = 18$. Although the answer 1309.79 does not meet the requirement of three decimal places, the student circles the final answer, 1310 gallons, which is correct, rounded to the nearest whole number as prompted in the question. In part (c) the student did not earn the first point for considering only one interior critical point. The second point was earned for naming the time at which the minimum occurs. The student does not show evidence of the absolute minimum and did not earn the third point. In part (d) the student writes the correct limits and sets up the equation correctly, earning both points.

Sample: 2B
Score: 4

The student earned 1 point in part (a), 2 points in part (b), and 1 point in part (d). Part (a) is correct. In part (b) the student earned the points for correct limits and correct integrand but did not earn the answer point, because the initial amount of 1200 gallons is not added. In part (c) the student does not address the question in any way. In part (d) the student earned the point for correct limits. The second point was not earned because the integral is equated to 0.

Question 3

Overview

The student is given a function describing the velocity of a particle at time t traveling along the *x*-axis and the position of the particle at time $t = 0$. Part (a) tests whether students can find the acceleration at a given time. Part (b) asks when the particle changes direction and when it is moving to the left. This requires the ability to determine and interpret the sign of the velocity. Part (c) asks for the position at time $t = 2$, requiring the evaluation of a definite integral of velocity as an accumulator expressing change in position and adding it to the initial position. Part (d) asks for average speed over an interval, requiring students to find the total distance traveled and to divide it by the elapsed time.

Sample: 3A
Score: 9

The student earned all 9 points. In part (b) the student earned the first point for $v(t) = 0$. The solutions are used in a labeled sign chart for $v(t)$. This chart establishes the fact that the particle changes direction at $t = 1$ and $t = 2$. The negative sign shows that the particle moves to the left when $1 < t < 2$.

Sample: 3B
Score: 6

The student earned 1 point in part (a), 2 points in part (b), and 3 points in part (c). Part (a) is correct. In part (b) the student earned the first point for $v(t) = 0$. The remark that the particle changes direction when $v(t)$ changes sign supports the answer that the direction changes at $t = 1$ and $t = 2$. The student does not recognize the particle travels to the left and did not earn the third point. In part (c) the student uses one step to evaluate the definite integral and another step to use the initial condition to find the correct answer. In part (d) an incorrect integrand is shown, so the student could not earn the answer point.

Sample: 3C
Score: 3

The student earned 1 point in part (a), 1 point in part (b), and 1 point in part (c). Part (a) is correct. In part (b) the student earned the first point for $v(t) = 0$. No evidence is presented to show that the direction changes at $t = 1$ and $t = 2$, so the second point was not earned. The interval is correct, but a reason is not provided. The student did not earn the third point. In part (c) the student does not set up a definite integral but gets a calculator-derived antiderivative. Evidence indicates that the antiderivative is evaluated at 0 and 2; thus the first point was earned. The initial condition is not used, and so the answer point was not earned. In part (d) the definite integral is not shown.

Question 5

Overview

Students are given an implicitly defined curve and asked to verify that the derivative is a specific function of x and y. This tests implicit differentiation. The solution is given so that students can work parts (b) and (c) without having to be successful in part (a). Part (b) tests whether students can combine the information in the equation of the curve with the formula for the derivative to find where the graph of the function has slope $\frac{1}{2}$. Part (c) pushes this a little further. Students have to observe that if the derivative is 0, then $y = 0,$ but that there is no point with y-coordinate 0 on the curve. Part (d) requires students to use the chain rule to find $\frac{dx}{dt}$ using their knowledge of $\frac{dy}{dx}$ and $\frac{dy}{dt}.$

Sample: 5A
Score: 9

The student earned all 9 points. In part (c) the student earned the $y = 0$ point because the numerator of the derivative is equated to 0. The explanation point was earned for explaining that substituting $y = 0$ in the curve results in a false statement. In part (d) the student earned the solves for x point by identifying $x = \frac{7}{3}$ when $y = 3,$ the chain rule point for correctly differentiating the curve with respect to time, and the answer point for correct evaluation.

Sample: 5B
Score: 6

The student earned 2 points in part (a), 2 points in part (b), 1 point in part (c), and 1 point in part (d). Parts (a) and (b) are correct. In part (c) the student earned the $y = 0$ point. The student does not explain why there are no points on the curve where the tangent to the curve is horizontal. As a result, the student did not earn the explanation point. In part (d) the student earned the chain rule point for correctly differentiating the curve with respect to time. However, the student never solves for x and instead uses the value of $t = 5$ for x. The student was not eligible for the answer point.

Sample: 5C
Score: 4

The student earned 2 points in part (a), 1 point in part (b), and 1 point in part (d). Part (a) is correct. In part (b) the student earned the point for setting the derivative equal to $\frac{1}{2}.$ Since only one of the points where the derivative is equal to $\frac{1}{2}$ is found, the student did not earn the answer point. The student did not earn any points in part (c). The student does not state that $y = 0$ and gives no explanation. In part (d) the student earned the implicit differentiation point. The student does not solve for x or give an answer.

Question 6

Overview

Students are given a separable differential equation with an initial condition. In part (a) they are asked to sketch the slope field at 12 points. In part (b) they need to know how to use the given formula for the derivative to find the equation of the line tangent to the graph of the solution through a specified point. In part (c) they are asked to solve the separable differential equation with the given initial condition.

Sample: 6A
Score: 9

The student earned all 9 points. When the student separates the variables in part (c), the $-\frac{1}{2}$ is moved to the dy side of the equation. This results in a slightly different, but correct, value for C.

Sample: 6B
Score: 6

The student earned 1 point in part (a) and 5 points in part (c). In part (a) two of the slope segments at $x = -1$ are incorrect. In part (b) the student's initial equation is correct, but an error is made in simplifying it. In part (c) the antiderivative for the left side is incorrect; however, the student successfully completes the work to finish the problem.

Sample: 6C
Score: 3

The student earned 2 points in part (a) and 1 point in part (b). In part (c) the student does not separate the variables properly and thus earned no points. A correct separation of variables groups the y-expressions with the dy and the x-expressions with the dx.