AP® Calculus AB
2005 Sample Student Responses

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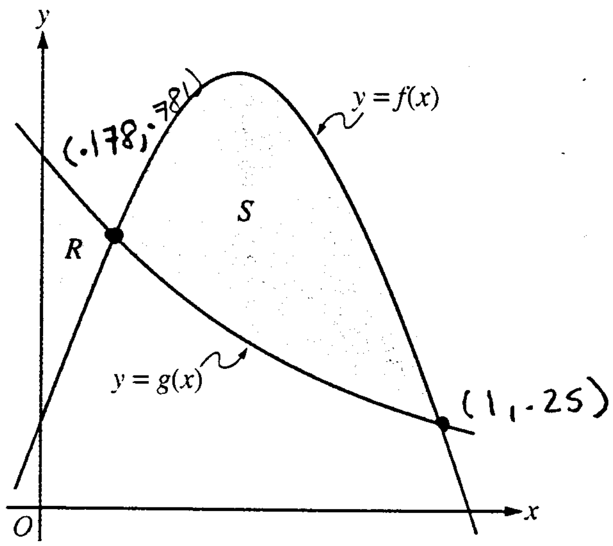
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$f(x) = \frac{1}{4} + \sin(\pi x)$

$g(x) = 4^{-x}$

$$A_R = \int_0^{.178} g(x) - f(x)\, dx$$

$$A_R = \int_0^{.178} 4^{-x} - \frac{1}{4} - \sin(\pi x)\, dx$$

$$\boxed{A_R = .0648\ u^2}$$

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Continue problem 1 on page 5.

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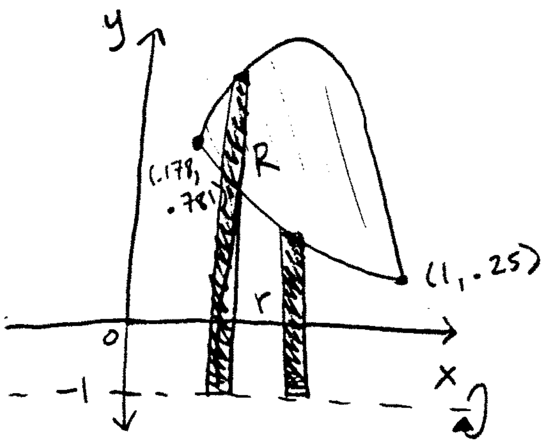
Work for problem 1(b)

$$A_S = \int_{.178}^{1} f(x) - g(x)\, dx$$

$$A_S = \int_{.178}^{1} \tfrac{1}{4} + \sin(\pi x) - 4^{-x}\, dx$$

$$\boxed{A_S = .410\ u^2}$$

Work for problem 1(c)



$R = f(x) - -1$

$r = g(x) - -1$

$$V_S = \int_{.178}^{1} \pi R^2 - \pi r^2\, dx$$

$$V_S = \int_{.178}^{1} \pi\left(\tfrac{1}{4} + \sin(\pi x) + 1\right)^2 - \pi\left(4^{-x} + 1\right)^2 dx$$

$$\boxed{V_S = 4.559\ u^3}$$

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B

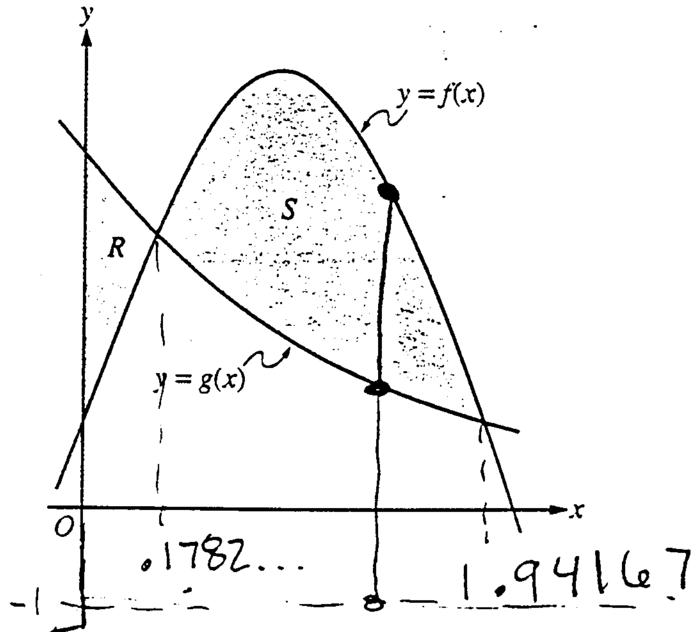
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



ork for problem 1(a)

$$\int_0^{.17821865} g(x) - f(x)\,dx$$

$$= .065$$

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Continue problem 1 on page 5.

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Work for problem 1(b)

$$\int_{.17821865}^{1.94167} f(x) - g(x)\, dx$$

$$= .117$$

Work for problem 1(c)

$$\int_{.17821805}^{1.94167} (-1-f(x))^2 - (-1-g(x))^2\, dx$$

$$= .618$$

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C

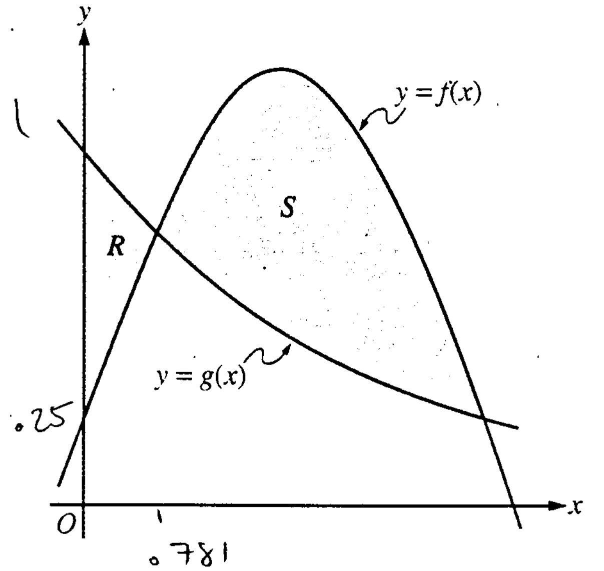
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$\int_0^{.781} 4^{-x} - \left[\frac{1}{4} + \sin(\pi x)\right] dx$

$\boxed{\text{Area of } R = -.2824 \text{ units}^2}$

via TI-83

itersect
.781

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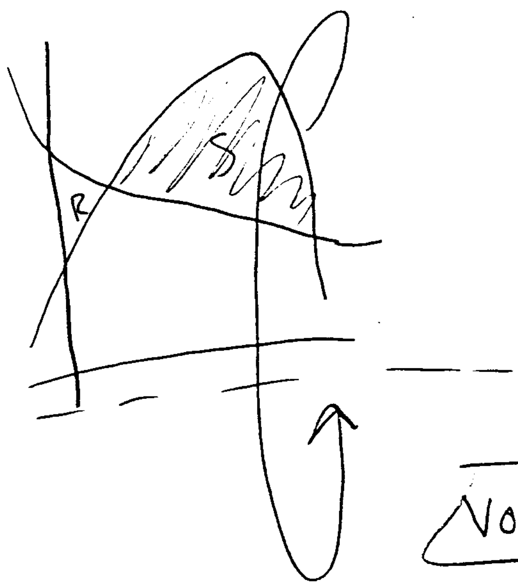
Work for problem 1(b)

$$\int_{.781}^{1} f(x) - g(x) = \int_{.781}^{1} \left[\frac{1}{4}+\sin(\pi x)\right] - 4^{-x}$$

$\boxed{\text{Area of } S = 0.0632 \text{ units}^2}$ ← via TI-83

Work for problem 1(c)



$$\pi \int_{-1}^{1} [(f(x)]^2 - [g(x)]^2 \, dx$$

$$\pi \int_{-1}^{1} \left(\frac{1}{4}\sin(\pi x)\right)^2 - 4^{-2x} \, dx$$

$\boxed{\text{Volume} = -14.5243 \text{ units}^3}$

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A

Work for problem 2(a)

$S = \int_0^6 R(t)\,dt = \boxed{31.816 \text{ yds}^3}$

Work for problem 2(b)

$Y(t) = \int_0^t (S(x) - R(x))\,dx + 2500$

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Continue problem 2 on page 7.

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A

Work for problem 2(c)

$Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = \boxed{-1.909 \ \frac{yds.^3}{hr.}}$

Work for problem 2(d)

Critical Number(s)

$Y'(t) = 0 \qquad t = 5.118 \qquad Y'(t): \; - \; | \; + \;$ at 5.118

Since Y' is negative to the left of 5.118 and positive to the right, there is a relative minimum at t=5.118. The absolute minimum can be this minimum or one of the end points.

$Y(0) = 2500 \qquad Y(5.118) = 2492.369 \qquad Y(6) = 2493.277$

The minimum of these values, and the minimum amount of sand on the beach, is at t = 5.118 and the value is 2492.369 yds³.

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B

Work for problem 2(a)

$\int_0^6 \left(2 + 5\sin\left(\frac{4\pi t}{25}\right)\right) dt = 31.816$ cubic yards

Work for problem 2(b)

$Y(t) = \int \left(\frac{15t}{1+3t} - \left(2 + 5\sin\left(\frac{4\pi t}{25}\right)\right)\right) dt$

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Work for problem 2(c)

$$\frac{15t}{1+3t} - 2 - 5\sin\left(\frac{4\pi t}{25}\right)$$

$$\frac{15(4)}{1+3(4)} - 2 - 5\sin\left(\frac{16\pi}{25}\right) = \frac{60}{13} - 2 - 4.524135262 =$$

$$-1.909$$

Work for problem 2(d)

$$\frac{15t}{1+3t} - 2 - 5\sin\left(\frac{4\pi t}{25}\right) = 0$$

$-$ | $+$

5.118

$t = 5.118$

5.118 is a minimum because the derivative of the function changes from negative to positive.

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Work for problem 2(a)

$R(t) = 2+5\sin\left(\frac{4\pi t}{25}\right)$

$\int_0^6 R(t)\,dt = 31.81593137$ yards of sand removed at $t=6$

Work for problem 2(b)

$Y(t) = \int_0^t R(t)\,dt + \int_0^t S(t)\,dt$

$Y(t) = \int_0^t \left(2+5\sin\left(\frac{4\pi t}{25}\right)\right)dt + \int_0^t \left(\frac{15t}{1+3t}\right)dt$

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Work for problem 2(c)

$t=4$

$R(4) = 2 + 5\sin\left(\frac{4\pi 4}{25}\right) = 6.524135262$

$S(4) = \frac{15(4)}{1+3(4)} = 4.615384615$

$-R(4) + S(4) = -1.908750647$ cubic yards/hr

Work for problem 2(d)

By the extreme value thm we are guaranteed an absolute min at either an end pt or critical pt since the function is continuous over the closed interval.

$R(t) + S(t) =$

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A

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

Work for problem 3(a)

$$T'(7) = \frac{55-62}{8-6} = -3.5 \ {}^\circ C/cm$$

Work for problem 3(b)

$$\text{Average Temp} = \frac{1}{8}\int_0^8 T(x)\,dx.$$

$$\text{Average} = \frac{1}{8}\cdot\Big[(100+93)(1)(\tfrac{1}{2}) + (93+70)(4)(\tfrac{1}{2}) + (62+70)(1)(\tfrac{1}{2}) + (55+62)(2)(\tfrac{1}{2})\Big]$$

$$= 75.688\ {}^\circ C$$

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Continue problem 3 on page 9.

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A

Work for problem 3(c)

$$\int_0^8 T'(x)\,dx = T(8) - T(0)$$
$$= 55 - 100$$
$$= -45\,^\circ C$$

$\int_0^8 T'(x)\,dx$ mean the total change (drop) in temperature of the wire from 0 cm to 8 cm.

Work for problem 3(d)

$T''(x) > 0 \Rightarrow T'(x)$ is increasing over the period.

from $x = 0$ to 1
slope $\Rightarrow -7$

$x = 1$ to 5
slope $\Rightarrow \frac{70-93}{5-1} = -5.75$

$x = 5$ to 6
slope $\Rightarrow \frac{62-70}{6-5} = -8.$

$x = 6$ to 8
slope $\Rightarrow \frac{55-62}{8-6} = -3.5$

By MVT

$\therefore$ between 5 to 6 there is a point with slope -8 which means a decrease of $T'(x)$

$\Downarrow$

$T'(x)$ is not always increasing

$\therefore T''(x) > 0$ is not consistent in the table data.

END OF PART A OF SECTION II

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B

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

Work for problem 3(a)

$$T'(7) \cong \frac{T(8)-T(6)}{8-6}$$

$$\cong \frac{55-62}{2}$$

$$\cong -\frac{7}{2}\ {}^\circ C/cm$$

Work for problem 3(b)

$$\text{Avg.} = \frac{1}{8}\int_0^8 T(x)\,dx$$

$$\cong \frac{1}{8}\left[\frac{1}{2}(T(0)+T(1)) + \frac{1}{2}(T(1)+T(5)) + \frac{1}{2}(T(5)+T(6)) + \frac{1}{2}(T(6)+T(8))\right]$$

$$\cong \frac{1}{8}\left[\frac{1}{2}(100+93) + \frac{1}{2}(93+70) + \frac{1}{2}(70+62) + \frac{1}{2}(62+55)\right]$$

$$\cong 37.813\ {}^\circ C$$

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B

Work for problem 3(c)

$$\int_0^8 T'(x)\,dx = T(8) - T(0)$$
$$= 55 - 100$$
$$= -45\,^\circ C$$

This is the total change in temperature of the wire, from one end to the other.

Work for problem 3(d)

$$T'(0.5) \cong \frac{T(1)-T(0)}{1-0}$$
$$\cong -7.00$$
$$T'(3) \cong \frac{T(5)-T(1)}{5-1}$$
$$\cong -5.750$$
$$T'(7) \cong -3.500$$

The table is consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$, since $T'(x)$ is increasing.

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C

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

Work for problem 3(a)

$$T'(7) \approx \frac{T(8) - T(6)}{8-6}$$

$$\approx \frac{55-62}{2}$$

$$\approx -7/2 \ °C/cm$$

Work for problem 3(b)

$$\text{Avg } T = \frac{\int_0^8 T(x)\,dx}{8}$$

$$\int_0^8 T(x)\,dx \approx \frac{b-a}{2n}\left[f(0) + 2f(1) + 2f(5) + 2f(6) + f(8)\right]$$

$$\approx \frac{8}{2(4)}\left[100 + 2(93) + 2(70) + 2(62) + 55\right]$$

$$\approx 605$$

$$\frac{\int_0^8 T(x)\,dx}{8} \approx \frac{605}{8} = \boxed{75.625 \ °C}$$

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c

Work for problem 3(c)

$$\int_0^8 T'(x)dx = T(x)\Big]_0^8$$
$$= T(8) - T(0)$$
$$= 55 - 100$$
$$= -45°C/cm$$

$\int_0^8 T'(x)dx$ represents the average rate of change of the temperature of the wire as x increases from 0 to 8.

Work for problem 3(d)

$T''(x) > 0$ implies that $T(x)$ is concave up, or that the rate of change is increasing.

The data in the table do not show that the rate of change is increasing from $x=0$ to $x=8$. For example: from $x=0$ to $x=1$, $T(x)$ decreases 7°C. In order for $T(x)$ to be concave up, it must decrease by less than 7°C/cm from $x=1$ to $x=5$: $\frac{T(5)-T(1)}{4} = -5.75°C$.

$T(x)$ decreases by 5.75°C, which less than 7°C, so it is changing at an increasing rate.

Therefore $T(x)$ is concave up and $T''(x)>0$ is true for $x=0$ to $x=5$

$\frac{T(6)-T(5)}{1} = -10$ ← This is not consistent however, so $T(x)$ is not concave up for all x in $0<x<8$

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NO CALCULATOR ALLOWED

A

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

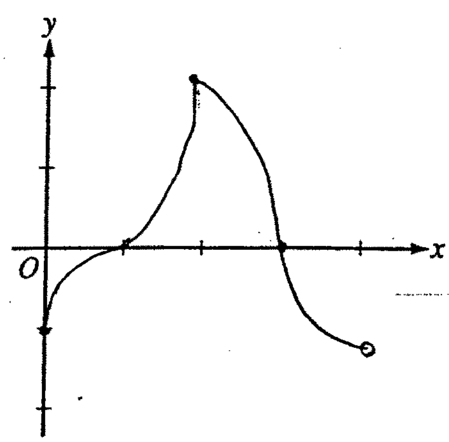
Work for problem 4(a)

f has one relative minimum at 0, where the end of the interval has not yet begun to increase.

There can be no relative extrema at 1 or 3 because the slope ($f'(x)$) does not change signs.

f has a relative maximum at 2, where slope ($f'(x)$) changes from positive to negative.

Work for problem 4(b)



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Work for problem 4(c)

~~g has a relative max at 0, at the end of the interval where g has not yet begun to decrease.~~

g has a relative minimum at 1, where slope (f) changes from negative to positive

g has a relative maximum at x = 3, where slope (f) changes from positive to negative.

Work for problem 4(d)

g has a point of inflection at x = 2, where g'' (f') changes from positive to negative.

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NO CALCULATOR ALLOWED

B

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$f'(x)$	4	+	0	+	$\emptyset$	−	−3	−
x	0		1		2		3	

f(x) has a relative maximum at x = 2 because it is increasing to the left and decreasing to the right.

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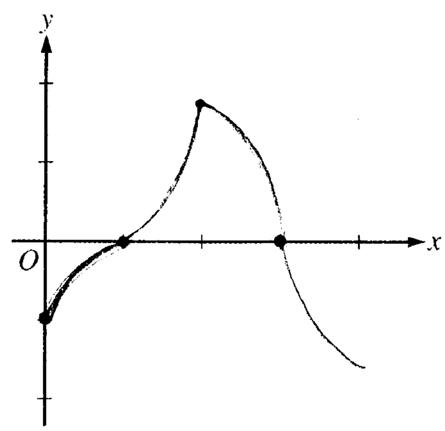
Work for problem 4(b)

$f''(x)$	−2	−	0	+	$\emptyset$	−	0	+
x	0		1		2		3	



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Work for problem 4(c)

$g(x) = \int_1^x f(t)\,dt \quad (0, 4)$

$g'(x) = f(x)$

$g'(x)$	-1	−	0	+	2	+	0	−
x	0		1		2		3	

g(x) has a relative minimum at x = 1 because it is decreasing to the left and increasing to the right and a relative maximum at x = 3 because it is increasing to the left and decreasing to the right.

Work for problem 4(d)

$g''(x) = f'(x)$

$g''(x)$	4	+	0	+	$\cancel{0}$	−	-3	−
x	0		1		2		3	

g(x) has a point of inflection at x = 2 because there is a sign change between the left and the right.

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C

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NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

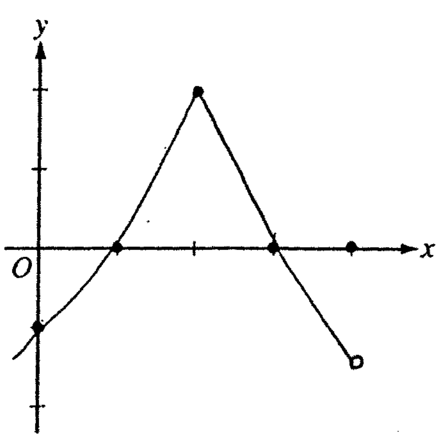
Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$x = 1$. This value is not a relative min or max because $f'(x)$ is not changing from positive to negative or from negative to positive at this point.

Work for problem 4(b)



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Work for problem 4(c)

$x = 1, 3$

$g'(x) = f(x)\,dx$

$g'(1) = 0$

$g'(3) = 0$

Rel. min: $x = 1$ $f'(x)$ goes from neg to pos

Rel. max: $x = 3$ $f'(x)$ goes from pos to neg

Work for problem 4(d)

$g''(x) = f'(x)\,dx$

$x = 1$

$g''(1) = f'(1) = 0$

this graph has a point of inflection at $x=1$ because its second derivative $= 0$

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A

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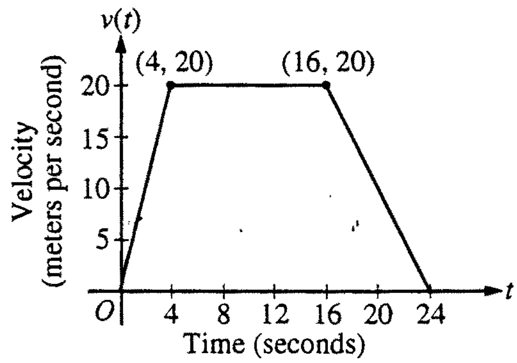

Work for problem 5(a)

$\int_0^{24} v(t)\,dt = \frac{1}{2}(12+24)(20) = \boxed{360 \text{ meters}}$

$\int_0^{24} v(t)\,dt$ is the displacement ~~of the~~ car in meters from time $t=0$ seconds to $t=24$ seconds - since the integral is positive, the car is 360 meters in the positive direction at time 24 seconds as compared with its position at time 0 seconds

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Work for problem 5(b)

$v'(4)$ does not exist because $\lim_{x \to 4^-} v'(x) = 5 \neq 0 = \lim_{x \to 4^+} v'(x)$

$v'(20) = \frac{-20}{8} = \boxed{\frac{-5}{2} \frac{\text{meters}}{\text{second}^2}}$

Continue problem 5 on page 13.

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Work for problem 5(c)

$$a(t) = \begin{cases} 5 \text{ m/s}^2 \text{ for } 0 \le t < 4 \text{ seconds} \\ 0 \text{ m/s}^2 \text{ for } 4 < t < 16 \text{ seconds} \\ -\frac{5}{2} \text{ m/s}^2 \text{ for } 16 < t \le 24 \text{ seconds} \end{cases}$$

* $a(t)$ undefined for $t = 4, 16$ seconds

Work for problem 5(d)

$$\frac{10-20}{20-8} = \frac{-10}{12} = \boxed{\frac{-5}{6} \frac{m}{s^2}}$$

The Mean Value Theorem does NOT guarantee a value for c, $8<c<20$, such that $v'(c)$ equals this average rate of change because $v(t)$ does not fulfill the requirements for the Mean Value Theorem since $v(t)$ is not differentiable on the interval $(8, 20)$ since $v'(t)$ is undefined at $t = 16$ seconds

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B

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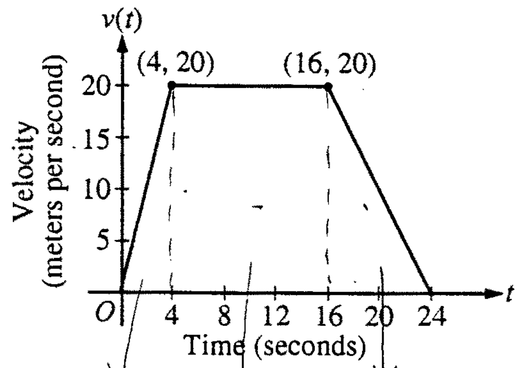

Work for problem 5(a)

$\frac{1}{2}(4)(20) + 12(20) + \frac{1}{2}(8)(20)$

$40 + 240 + 80 = 360 = \int_0^{24} v(t)\,dt$

$\int_0^{24} v(t)\,dt = 360$ meters which is the total distance the car traveled from $t=0$ to $t=24$.

Work for problem 5(b)

The derivative of v at $t=4$ does not exist because it is located at a corner.

$v'(20) = \frac{20-0}{16-24} = \frac{20}{-8} = -\frac{5}{2}\ \frac{\text{meters}}{\text{sec}}$

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Continue problem 5 on page 13.

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Work for problem 5(c)

slope of $v(t)$ from $t=0$ to $t=4$: 5
slope " $t=4$ to $t=16$: 0
slope " $t=16$ to $t=24$: $-\frac{5}{2}$

$$a(t) = \begin{cases} 5, & 0 \le x \le 4 \\ 0, & 4 < x \le 16 \\ -5/2, & 16 < x \le 24 \end{cases}$$

Work for problem 5(d)

The Mean Value Theorem does not guarantee this because $v(t)$ is not differentiable over $8 \le t \le 20$.

$$\cancel{\frac{\int [v(20) - v(8)] dv}{20-8} = \frac{\int (10-20) dv}{12} = \frac{\int -10 dv}{12}}$$

$4(20) + 4(20) + \frac{1}{2}(4)(20+10) = 80 + 80 + 60 = 220$

$\frac{220}{12} \frac{\text{meters}}{\text{sec}^2}$

160
60
220

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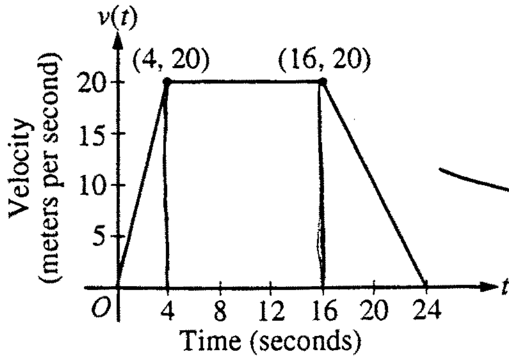

Work for problem 5(a)

$\int_0^{24} (v(t))\,dt$ is asking for the total distance traveled in these 24 seconds. This is the total Area underneath the graph.

$A = \frac{1}{2}(4)(20) + 12(20) + \frac{1}{2}(8)(20)$

$= 40 + 240 + 80$

$= 360$

Work for problem 5(b)

$v'(t)$ is asking for the slope at a certain point in time. A $t=4$ and $t=20$ the slope is undefined because there is a corner at both of these times.

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Work for problem 5(c)

$A(t) = v'(t)$

$$v(t) = \begin{cases} 5x & 0 \le x \le 4 \\ 20 & 4 < x \le 16 \\ -\frac{5}{2}x + 60 & 16 < x \le 24 \end{cases}$$

$$\boxed{A(t) = \begin{cases} 5 & 0 \le x \le 4 \\ 0 & 4 < x \le 16 \\ -\frac{5}{2} & 16 < x \le 24 \end{cases}}$$

← 5 is the deriv of 5x

← 0 is the deriv of 20

← $-\frac{5}{2}$ is the deriv of $-\frac{5}{2}x + 60$

Work for problem 5(d)

avg rate of chg $= \frac{f(b) - f(a)}{b - a} = \frac{f(20) - f(8)}{20 - 8} = \frac{10 - 20}{20 - 8} = \boxed{-\frac{5}{6}}$

• yes C has a garunteed value because v(t) is continuous from $8 \le x \le 20$.

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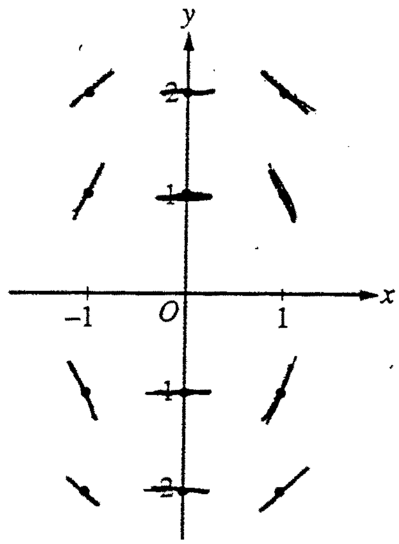
A

Work for problem 6(a)



Work for problem 6(b)

line tangent: $y+1 = 2(x-1)$

$f(x) = 2(x-1) - 1$

$f(1.1) = 2(1.1-1) - 1$

$f(1.1) = .2 - 1 = -.8$

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Work for problem 6(c)

$$\frac{dy}{dx} = \frac{-2x}{y}$$

$$\int y\,dy = \int -2x\,dx$$

$$\frac{y^2}{2} = -x^2$$

$$y^2 = -2x^2 + C$$

$$y = \pm\sqrt{-2x^2+C}$$

$$-1 = -\sqrt{-2(1)^2+C}$$

$$1 = -2 + C$$

$$C = 3$$

$$y = -\sqrt{-2x^2+3}$$

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END OF EXAM

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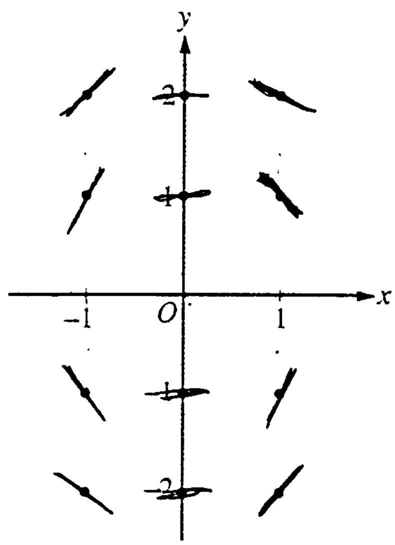
B

Work for problem 6(a)

$\frac{dy}{dx} = -\frac{2x}{y}$ 0

$\frac{-2}{1}$ $\frac{-2}{-1}$

$\frac{+2}{1}$ $\frac{2}{-1}$



Work for problem 6(b)

$y+1 = 2(x-1) = 2x-2$

$y = 2x - 3$

$y = 2(1.1) - 3$

$= 2.2 - 3$

$f(1.1) = -.8$

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 B

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Work for problem 6(c)

$\frac{dy}{dx} = -\frac{2x}{y}$

$\int y\,dy = \int -2x\,dx$

$\frac{y^2}{\cancel{2}} = -\frac{\cancel{2}x^2}{\cancel{2}}$

$y^2 = -2x^2$

$y =$

END OF EXAM

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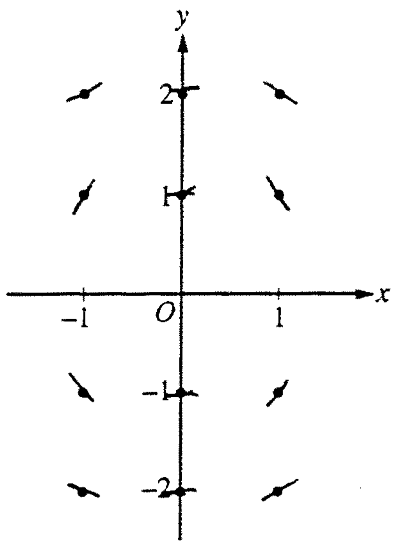
Work for problem 6(a)



Work for problem 6(b)

$y+1 = 2(x-1)$

$f(x) = 2x - 1$

$f(1.1) = 2.2 - 1$

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Work for problem 6(c)

$$\frac{dy}{dx} = \frac{-2x}{y}$$

$$dy\ y = -2x\,dx$$

$$1y = -2x$$

$$f(x) = -2x$$

$$f(1) = 2$$

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END OF EXAM

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