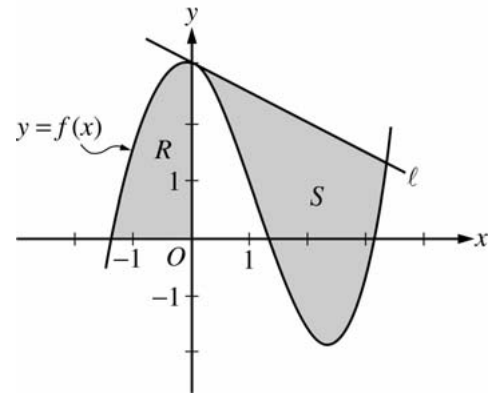


AP[®] CALCULUS AB
2006 SCORING GUIDELINES (Form B)

Question 1

Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f , and let S be the shaded region bounded by the graph of f and line ℓ , the line tangent to the graph of f at $x = 0$, as shown above.



- Find the area of R .
- Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
- Write, but do not evaluate, an integral expression that can be used to find the area of S .

For $x < 0$, $f(x) = 0$ when $x = -1.37312$.
 Let $P = -1.37312$.

(a) Area of $R = \int_P^0 f(x) dx = 2.903$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Volume = $\pi \int_P^0 ((f(x) + 2)^2 - 4) dx = 59.361$

4 : $\begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) The equation of the tangent line ℓ is $y = 3 - \frac{1}{2}x$.

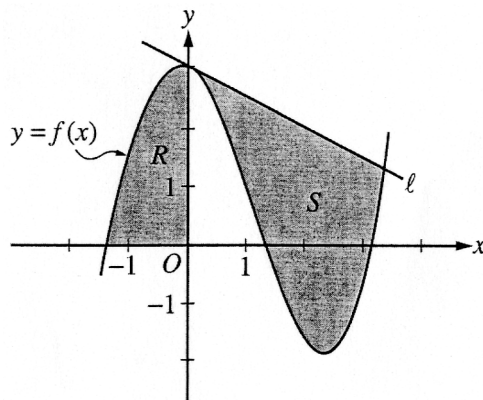
The graph of f and line ℓ intersect at $A = 3.38987$.

Area of $S = \int_0^A \left(\left(3 - \frac{1}{2}x \right) - f(x) \right) dx$

3 : $\begin{cases} 1 : \text{tangent line} \\ 1 : \text{integrand} \\ 1 : \text{limits} \end{cases}$

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$\text{Area } R = \int_a^b f(x) \, dx$$

$$f(x) = 0 \Rightarrow \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x = 0$$

$$x = a = -1.373$$

$$b = 0$$

$$R = \int_{-1.373}^0 f(x) \, dx = 2.903$$

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Continue problem 1 on page 5.

Work for problem 1(b)

$$V = \pi \int_{-1.373}^0 \left[(f(x)+2)^2 - (2)^2 \right] dx = 59.361 \text{ unit}^3$$

Work for problem 1(c)

$$f'(x) = \frac{3x^2}{4} - \frac{2x}{3} - \frac{1}{2} - 3\sin x$$

$$f'(0) = \text{slope at } x=0$$

$$f'(0) = -\frac{1}{2}$$

$$f(0) = 3$$

$$y - 3 = -\frac{1}{2}(x - 0)$$

$$y = 3 - \frac{x}{2}$$

intersection point when $3 - \frac{x}{2} = f(x) \Rightarrow x = 3.390$

$$\text{Area } S = \int_0^{3.390} \left[3 - \frac{x}{2} - \left(\frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x \right) \right] dx$$

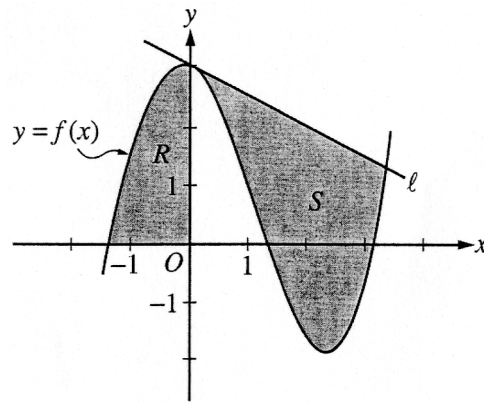
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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$\frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x = 0$$

$$x = -1.37312$$

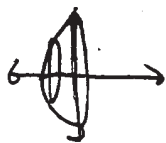
$$\int_{-1.37312}^0 \left(\frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x \right) dx$$

$$= 2.90309$$

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Continue problem 1 on page 5.

Work for problem 1(b)



$$r = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$$

$$\int_{-1.37312}^0 \pi r^2 dx = \int_{-1.37312}^0 \pi \left(\frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x \right)^2 dx = 22.880$$

Work for problem 1(c)

② $f'(0) = -\frac{1}{2}$ $y = -\frac{1}{2}x + 3$

$y = -\frac{1}{2}x + 3$ TSL

Intersect $-\frac{1}{2}x + 3 = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$

$x = 3.38987$ $y = 1.30507$

$$\int_0^{3.38987} \left| \left(-\frac{1}{2}x + 3 \right) - \left(\frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x \right) \right| dx$$

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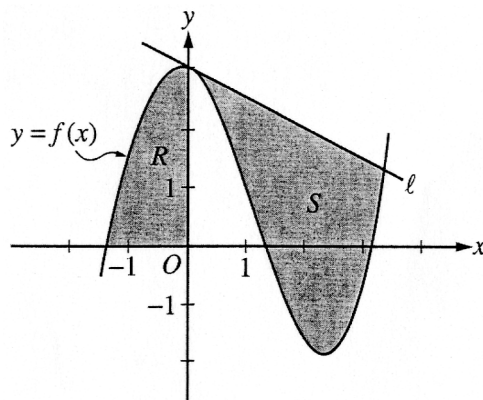
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CALCULUS BC
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

bounds extend from
-1.37312 to 0

$$A_R = \int_{-1.37312}^0 \left(\frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x \right) dx$$

$$= 2.903$$

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Continue problem 1 on page 5.

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Work for problem 1(b)

$$\pi \int_{-1.373/2}^0 \left(\frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x \right)^2 - 2^2$$

$$= 1.79\pi$$

Work for problem 1(c)

$$A_s = \int_0^{3.1} 1 - f(x) dx$$

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AP[®] CALCULUS AB
2006 SCORING COMMENTARY (Form B)

Question 1

Overview

This problem presented students with two regions. Region R was bounded in the second quadrant by a graph and the two axes. Region S was bounded by the graph and the line tangent to the graph at one point. Students needed to use integration to find two areas and a volume. In order to answer parts (a) and (b), students also had to find a zero of the function to obtain bounding values for region R . Part (a) asked students to find the area of R . Part (b) asked students to find the volume of the solid generated by rotating R about a horizontal line. In part (c) students had to find the equation of the tangent line and the x -coordinate of a point of intersection of the line and the graph in order to write an integral expression for the area of S .

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), and 3 points in part (c). The work in part (a) is correct. In part (b) the student earned the limits and constant point. The student writes an integral for rotation about the x -axis and does not consider the horizontal line $y = -2$. Because of this error in the integrand, the student was not eligible for the answer point. The work in part (c) earned all 3 points.

Sample: 1C
Score: 4

The student earned 4 points: 2 points in part (a) and 2 points in part (b). The work in part (a) is correct. In part (b) the student earned the limits and constant point. The student earned 1 of the 2 integrand points. The first term of the integrand is incorrect since 2 was not added to $f(x)$. Because of this error in the integrand, the student was not eligible for the answer point. In part (c) the equation of the tangent line is not found, so the tangent line point was not earned. Since an equation for a tangent line was not found, the student could not earn the integrand point. In addition, the student did not earn the limits point for estimating the intersection point from the given graph.