## Question 6

The twice-differentiable function $f$ is defined for all real numbers and satisfies the following conditions:
$f(0) = 2$, $f'(0) = -4$, and $f''(0) = 3$.

(a) The function $g$ is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where $a$ is a constant. Find $g'(0)$ and $g''(0)$ in terms of $a$. Show the work that leads to your answers.

(b) The function $h$ is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where $k$ is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of $h$ at $x = 0$.

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(a) $g'(x) = ae^{ax} + f'(x)$
$g'(0) = a - 4$

$g''(x) = a^2 e^{ax} + f''(x)$
$g''(0) = a^2 + 3$

$4 : \begin{cases} 1 : g'(x) \\ 1 : g'(0) \\ 1 : g''(x) \\ 1 : g''(0) \end{cases}$

(b) $h'(x) = f'(x)\cos(kx) - k\sin(kx) f(x)$
$h'(0) = f'(0)\cos(0) - k\sin(0) f(0) = f'(0) = -4$
$h(0) = \cos(0) f(0) = 2$
The equation of the tangent line is $y = -4x + 2$.

$5 : \begin{cases} 2 : h'(x) \\ 3 : \begin{cases} 1 : h'(0) \\ 1 : h(0) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$

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Work for problem 6(a)

$g(x) = e^{ax} + f(x)$

$g'(x) = a.e^{ax} + f'(x) \Rightarrow g'(0) = a.e^0 + f'(0)$

$= a - 4$

$g''(x) = a^2 e^{ax} + f''(x) \Rightarrow g''(0) = a^2.e^0 + f''(0)$

$= a^2 + 3$

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NO CALCULATOR ALLOWED

Work for problem 6(b)

$h(x) = \cos(kx) f(x)$

$h'(x) = -\sin(kx).k.f(x) + \cos(kx).f'(x)$

$= -k.\sin(kx).f(x) + \cos(kx).f'(x)$

$h'(0) = -k.\sin 0.f(0) + \cos(0).f'(0)$

$= 0 + 1.(-4) = -4$

$h(0) = \cos 0.f(0) = 1.a = a$

$y = mx + b$

$y = -4x + b$

$a = -4.0 + b$

$b = a$

the line tangent to the graph of $h$ is $y = -4x + a$

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**STOP**

**END OF EXAM**

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**NO CALCULATOR ALLOWED**

Work for problem 6(a)

$u = ax$
$du = a$

$g(x) = e^{ax} + f(x)$

$g'(x) = ae^{ax} + f'(x)$

$g'(0) = ae^{a0} + 4$

$g'(0) = a + 4$

$g''(0) = 2ae^{ax} + f''(0)$

$g''(0) = 2ae^{a(0)} + 3$

$g''(0) = 2a + 3$

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Work for problem 6(b)

$h(x) = \cos(kx) f(x)$

$h'(x) = \cos(kx) f'(x) + k\sin(kx) f(x)$

$h'(0) = \cos(0) \cdot -4 + k\sin(0)\, 2$

$h'(0) = -4 + 0$

$h'(0) = -4$

$h(0) = \cos(k0) f(0)$

$h(0) = 1 \cdot 2$

$h(0) = 2$

$y - 2 = -4(x - 0)$

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**END OF EXAM**

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**NO CALCULATOR ALLOWED**

Work for problem 6(a)

$f(0)=2 \quad f'(0)=-4 \quad f''(0)=3$

$g(x)=e^{ax}+f(x)$

$g'(x)=e^{ax}\cdot a+f'(x)$

$e^x \frac{d}{dx}=e^x$

$e^{f(x)} \frac{d}{dx}=e^{f(x)} f(x) \frac{d}{dx}$

$g'(x)=ae^{ax}+-4$

$g'(x)=ae^{ax}-4$

$e^0=1$ $g'(0)=a e^{a(0)}-4$

$g'(0)=a-4$

$g''(x)=[ae^{ax}\cdot a]$

$g''(x)=a^2e^{ax}$

$g''(0)=a^2(e^{a(0)}=1)$

$g''(0)=a^2$

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Work for problem 6(b)

$h(x) = \cos(kx)\, f(x)$

$h'(x) = -\sin(kx) \cdot k \cdot f'(x)$

$h'(x) = -k f(x) \sin(kx)$ (1)

$h(x) = \cos(k(0))\, f(0)$

$= \cos 0 \cdot 2$

$= 2$

point $(0, 2)$

$y - 2 = (-k f(x) \sin(kx))(x - 0)$

$y = [-k f(x) \sin(kx)](x) + 2$

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**END OF EXAM**

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# AP® CALCULUS AB
# 2006 SCORING COMMENTARY

## Question 6

### Overview

This problem gave students the values of $f(0)$, $f'(0)$, and $f''(0)$ for a twice-differentiable function $f$. In part (a) the function $g$ was defined as the sum of $f$ and an exponential function involving a parameter. Students had to use the chain rule and addition rule for differentiation, and the given information about $f$, to compute $g'(0)$ and $g''(0)$ in terms of that parameter. Part (b) introduced a function $h$ as the product of $f$ and a cosine function involving the parameter $k$. Here students had to use the chain rule and product rule to compute the derivative of $h$ and then use that derivative to write an equation for the line tangent to the graph of $h$ at $x = 0$. Although not asked, it was hoped that the students would make the interesting observation that the equation of the tangent line at $x = 0$ is the same for all values of the parameter $k$.

### Sample: 6A
### Score: 9

The student earned all 9 points.

### Sample: 6B
### Score: 6

The student earned 6 points: 2 points in part (a) and 4 points in part (b). In part (a) the student correctly presents $g'(x)$ and $g'(0)$. The student presents an incorrect $g''(x)$ and was not eligible for the fourth point in part (a). In part (b) the student's $h'(x)$ includes a sign error and earned only 1 of the 2 derivative points. The presented value for $h(0)$ is correct, $h'(0)$ is consistent with the student's $h'(x)$, and the student correctly writes an equation of the tangent line.

### Sample: 6C
### Score: 3

The student earned 3 points: 2 points in part (a) and 1 point in part (b). In part (a) the student correctly presents $g'(x)$ and $g'(0)$. The student presents an incorrect $g''(x)$ and was not eligible for the fourth point in part (a). In part (b) the student presents an incorrect $h'(x)$. The $h(0)$ point was earned. The student does not find $h'(0)$ and does not write an equation of the tangent line.

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