

AP[®] CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

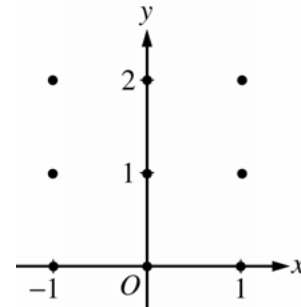
- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)

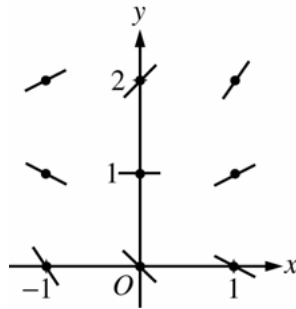
- (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Describe the region in the xy -plane in which all solution curves to the differential equation are concave up.

- (c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

- (d) Find the values of the constants m and b , for which $y = mx + b$ is a solution to the differential equation.



(a)



(b) $\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx} = \frac{1}{2}x + y - \frac{1}{2}$

Solution curves will be concave up on the half-plane above the line

$$y = -\frac{1}{2}x + \frac{1}{2}.$$

(c) $\left. \frac{dy}{dx} \right|_{(0,1)} = 0 + 1 - 1 = 0$ and $\left. \frac{d^2y}{dx^2} \right|_{(0,1)} = 0 + 1 - \frac{1}{2} > 0$

Thus, f has a relative minimum at $(0, 1)$.

- (d) Substituting $y = mx + b$ into the differential equation:

$$m = \frac{1}{2}x + (mx + b) - 1 = \left(m + \frac{1}{2}\right)x + (b - 1)$$

Then $0 = m + \frac{1}{2}$ and $m = b - 1$: $m = -\frac{1}{2}$ and $b = \frac{1}{2}$.

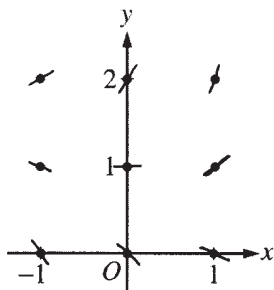
2 : Sign of slope at each point and relative steepness of slope lines in rows and columns.

3 : $\begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{description} \end{cases}$

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

2 : $\begin{cases} 1 : \text{value for } m \\ 1 : \text{value for } b \end{cases}$

Work for problem 5(a)



Work for problem 5(b)

$$\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx} = \frac{1}{2}x + y - \frac{1}{2}$$

curves are concave up : $\frac{d^2y}{dx^2} > 0$

$$\frac{1}{2}x + y - \frac{1}{2} > 0$$

$$\frac{1}{2}x + y > \frac{1}{2}$$

$$x + 2y > 1$$

$x + 2y - 1 > 0$ of the solution curves
 ↑ when coordinates satisfy this condition,
 the [^]curves are concave up.
 solution

$$\dots x \geq 0 \text{ and } y > \frac{1}{2}$$

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Continue problem 5 on page 13.

Work for problem 5(c)

$$f(0) = 1$$

$$\frac{dy}{dx} = \frac{1}{2} \cdot 0 + 1 - 1 = 0$$

$$\frac{d^2y}{dx^2} = \frac{1}{2} \cdot 0 + 1 - \frac{1}{2} = \frac{1}{2} > 0$$

f has a relative minimum at $x = 0$

as $\frac{dy}{dx}$ attains zero and change its sign from negative to positive.

Work for problem 5(d)

$$y = mx + h$$

$$\frac{dy}{dx} = m = \frac{1}{2}x + y - 1 \rightarrow m = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = 0 = \frac{1}{2}x + y - \frac{1}{2} \rightarrow \frac{1}{2}x + y = \frac{1}{2}$$

$$\begin{cases} y = -\frac{1}{2}x + h \rightarrow y = -x + 2h \\ \frac{1}{2}x + y = \frac{1}{2} \rightarrow x + y = 1 \rightarrow x - x + 2h = 1 \\ h = \frac{1}{2} \end{cases}$$

$$m = -\frac{1}{2}, \quad h = \frac{1}{2}$$

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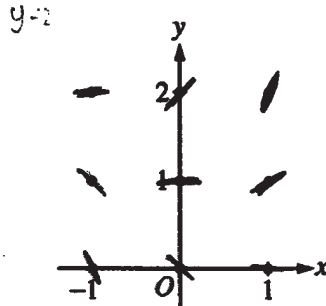
5

AB

NO CALCULATOR ALLOWED

5B₁

Work for problem 5(a)



Work for problem 5(b)

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} \left(\frac{1}{2}x + y - 1 \right) = \frac{1}{2} + \frac{dy}{dx} = \frac{1}{2}x + y - \frac{1}{2}$$

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Work for problem 5(c)

If $f(0) = 1$, when $x = 0, y = 1$.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x + y - 1 = \frac{1}{2} \cdot 0 + 1 - 1 = 0$$

$\therefore f$ have neither relative maximum and relative minimum values.

Work for problem 5(d)

$$y = mx + b$$

$$y = -\frac{1}{2}x + b$$

$$\frac{dy}{dx} = m = \frac{1}{2}x + y - 1 = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = 0 = \frac{1}{2}x + y - \frac{1}{2} = m + \frac{1}{2} \quad \therefore m = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + b \Rightarrow b = \frac{1}{2}x + y = m + 1 = -\frac{1}{2} + 1 = \frac{1}{2} \quad \therefore b = \frac{1}{2}$$

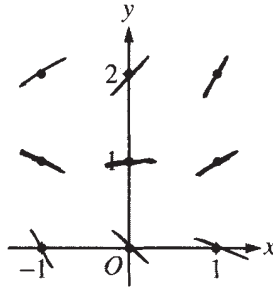
$$\therefore y = -\frac{1}{2}x + \frac{1}{2} \quad (m = -\frac{1}{2}, b = \frac{1}{2})$$

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Work for problem 5(a)



Work for problem 5(b)

$$\frac{d}{dx} \left(\frac{1}{2}x + y - 1 \right) = \frac{1}{2} + \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}x + y - \frac{1}{2}$$

The first quadrant $(x+, y+)$ (excluding the origin) is all concave up

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Work for problem 5(c)

neither

At $x = \emptyset$ the slope has a range from neg to pos

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Work for problem 5(d)

$$y = \left(\frac{1}{2}x + y - 1\right)x + b$$

$$y = \frac{1}{2}x^2 + xy - x + b$$

-xy -xy

$$\frac{y(1-x)}{1-x} = \frac{\frac{1}{2}x^2 - x + b}{1-x}$$

$$y = \frac{\frac{1}{2}x^2 - x + b}{1-x}$$

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AP[®] CALCULUS AB
2007 SCORING COMMENTARY (Form B)

Question 5

Sample: 5A

Score: 9

The student earned all 9 points.

Sample: 5B

Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). Correct work is presented in parts (a) and (d). In part (b) the student earned the first 2 points but gives no description of the region, so the third point was not earned. In part (c) the student does not conclude a relative minimum or provide a justification, so no points were earned. The student finds $\frac{dy}{dx}$ but does not calculate the second derivative at $(0, 1)$ to determine the concavity of the graph.

Sample: 5C

Score: 4

The student earned 4 points: 2 points in part (a), 2 points in part (b), no points in part (c), and no points in part (d). Correct work is presented in part (a). In part (b) the student earned the first 2 points but gives an incorrect description of the region, so the third point was not earned. There is no relevant work provided for part (c). In part (d) the student does not find the values of m or b .