CollegeBoard
Advanced Placement Program

AP® Calculus AB
2008 Scoring Guidelines
Form B

Question 1

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

The graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$ intersect at the points $(0, 0)$ and $(9, 3)$.	
(a) $\int_0^9 \left(\sqrt{x} - \frac{x}{3} \right) dx = 4.5$ OR $\int_0^3 \left(3y - y^2 \right) dy = 4.5$	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $\pi \int_0^3 \left((3y+1)^2 - \left(y^2 + 1 \right)^2 \right) dy$ $= \frac{207\pi}{5} = 130.061 \text{ or } 130.062$	$4 : \begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\int_0^3 \left(3y - y^2 \right)^2 dy = 8.1$	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

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Question 2

For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

(a) $\int_0^2 r(t)\,dt = 206.370$ kilometers

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{dg}{dt} = \dfrac{dg}{dx} \cdot \dfrac{dx}{dt}; \quad \dfrac{dx}{dt} = r(t)$

$\left.\dfrac{dg}{dt}\right|_{t=2} = \left.\dfrac{dg}{dx}\right|_{x=206.370} \cdot r(2)$

$= (0.050)(120) = 6$ liters/hour

$3 : \begin{cases} 2 : \text{uses chain rule} \\ 1 : \text{answer with units} \end{cases}$

(c) Let T be the time at which the car's speed reaches 80 kilometers per hour.

Then, $r(T) = 80$ or $T = 0.331453$ hours.

At time T, the car has gone

$x(T) = \int_0^T r(t)\,dt = 10.794097$ kilometers

and has consumed $g(x(T)) = 0.537$ liters of gasoline.

$4 : \begin{cases} 1 : \text{equation } r(t) = 80 \\ 2 : \text{distance integral} \\ 1 : \text{answer} \end{cases}$

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Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

(a) $\frac{(0+7)}{2}\cdot 8 + \frac{(7+8)}{2}\cdot 6 + \frac{(8+2)}{2}\cdot 8 + \frac{(2+0)}{2}\cdot 2$
$= 115 \text{ ft}^2$

1 : trapezoidal approximation

(b) $\frac{1}{120}\int_0^{120} 115v(t)\,dt$
$= 1807.169 \text{ or } 1807.170 \text{ ft}^3/\text{min}$

3 : $\begin{cases} 1 : \text{limits and average value constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{24} 8\sin\left(\frac{\pi x}{24}\right)dx = 122.230 \text{ or } 122.231 \text{ ft}^2$

2 : $\begin{cases} 1 : \text{integra1} \\ 1 : \text{answer} \end{cases}$

(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is
$\frac{1}{20}\int_{40}^{60} C \cdot v(t)\,dt = 2181.912 \text{ or } 2181.913 \text{ ft}^3/\text{min}.$

Yes, water must be diverted since the average volumetric flow for this 20-minute period exceeds 2100 ft^3/min.

3 : $\begin{cases} 1 : \text{volumetric flow integral} \\ 1 : \text{average volumetric flow} \\ 1 : \text{answer with reason} \end{cases}$

Question 4

The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4 + t^2}\, dt$ and $g(x) = f(\sin x)$.

(a) Find $f'(x)$ and $g'(x)$.

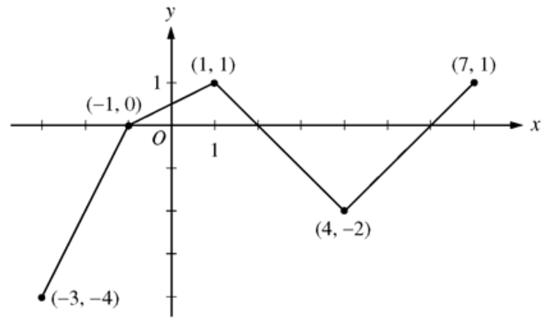
(b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.

(c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \le x \le \pi$. Justify your answer.

(a) $f'(x) = 3\sqrt{4 + (3x)^2}$ $g'(x) = f'(\sin x) \cdot \cos x$ $= 3\sqrt{4 + (3\sin x)^2} \cdot \cos x$	$4 : \begin{cases} 2 : f'(x) \\ 2 : g'(x) \end{cases}$
(b) $g(\pi) = 0,\ g'(\pi) = -6$ Tangent line: $y = -6(x - \pi)$	$2 : \begin{cases} 1 : g(\pi) \text{ or } g'(\pi) \\ 1 : \text{tangent line equation} \end{cases}$
(c) For $0 < x < \pi$, $g'(x) = 0$ only at $x = \frac{\pi}{2}$. $g(0) = g(\pi) = 0$ $g\left(\frac{\pi}{2}\right) = \int_0^3 \sqrt{4 + t^2}\, dt > 0$ The maximum value of g on $[0, \pi]$ is $\int_0^3 \sqrt{4 + t^2}\, dt$.	$3 : \begin{cases} 1 : \text{sets } g'(x) = 0 \\ 1 : \text{justifies maximum at } \frac{\pi}{2} \\ 1 : \text{integral expression for } g\left(\frac{\pi}{2}\right) \end{cases}$

Question 5

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.



Graph of g'

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

(a) g' changes from increasing to decreasing at $x = 1$;
g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at $x = 1$ and $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) The only sign change of g' from positive to negative in the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x)\,dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$
$$g(2) = 5$$
$$g(7) = 5 + \int_2^{7} g'(x)\,dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \le x \le 7$ is $\frac{15}{2}$.

$3 : \begin{cases} 1 : \text{identifies } x = 2 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{maximum value and justification} \end{cases}$

(c) $$\frac{g(7) - g(-3)}{7 - (-3)} = \frac{\frac{3}{2} - \frac{15}{2}}{10} = -\frac{3}{5}$$

$2 : \begin{cases} 1 : \text{ difference quotient} \\ 1 : \text{ answer} \end{cases}$

(d) $$\frac{g'(7) - g'(-3)}{7 - (-3)} = \frac{1 - (-4)}{10} = \frac{1}{2}$$

No, the MVT does *not* guarantee the existence of a value c with the stated properties because g' is not differentiable for at least one point in $-3 < x < 7$.

$2 : \begin{cases} 1 : \text{ average value of } g'(x) \\ 1 : \text{ answer “No” with reason} \end{cases}$

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Question 6

Consider the closed curve in the *xy*-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the *x*-axis? Explain your reasoning.

(a) $2x + 2 + 4y^3\dfrac{dy}{dx} + 4\dfrac{dy}{dx} = 0$ $\left(4y^3+4\right)\dfrac{dy}{dx} = -2x - 2$ $\dfrac{dy}{dx} = \dfrac{-2(x+1)}{4\left(y^3+1\right)} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$	$2: \begin{cases} 1: \text{implicit differentiation} \\ 1: \text{verification} \end{cases}$
(b) $\left.\dfrac{dy}{dx}\right\|_{(-2,\,1)} = \dfrac{-(-2+1)}{2(1+1)} = \dfrac{1}{4}$ Tangent line: $y = 1 + \dfrac{1}{4}(x+2)$	$2: \begin{cases} 1: \text{slope} \\ 1: \text{tangent line equation} \end{cases}$
(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$. On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$. Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.	$3: \begin{cases} 1: y = -1 \\ 1: \text{substitutes } y = -1 \text{ into the equation of the curve} \\ 1: \text{answer} \end{cases}$
(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$. The curve crosses the *x*-axis where $y = 0$. $(-1)^2 + 2(-1) + 0^4 + 4 \cdot 0 \neq 5$ No, the curve cannot have a horizontal tangent where it crosses the *x*-axis.	$2: \begin{cases} 1: \text{works with } x = -1 \text{ or } y = 0 \\ 1: \text{answer with reason} \end{cases}$