CollegeBoard
Advanced Placement Program

Student Performance Q&A:

2008 AP® Calculus AB and Calculus BC Free-Response Questions

The following comments on the 2008 free-response questions for AP® Calculus AB and Calculus BC were written by the Chief Reader, Michael Boardman of Pacific University in Forest Grove, Oregon. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student performance in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question AB1/BC1

What was the intent of this question?

In this problem, students were given the graph of a region R bounded by two curves in the xy-plane. The points of intersection of the two curves were observable from the supplied graph. The formulas for the curves were given—a trigonometric function and a cubic polynomial—and students needed to match the appropriate functions to the upper and lower bounding curves. In each part, students had to set up and evaluate an appropriate integral. Part (a) asked for the area of R. Part (b) asked for the area of the portion of R below the line $y = -2,$ so students needed to use a calculator to solve for the x-coordinates of the points of intersection of $y = -2$ and the lower curve to set up the appropriate integral. Part (c) asked for the volume of a solid with base R whose cross sections perpendicular to the x-axis are squares. In part (d) students were asked to find a volume in an applied setting. They had to determine that cross sections perpendicular to the x-axis are rectangles with one dimension in region R and the other dimension supplied by $h(x) = 3 - x.$

How well did students perform on this question?

Students who recognized that the area of the region R was independent of the region's placement relative to the x-axis did very well on this problem. Students who believed that the region had to be broken up into smaller regions, each of whose areas had to be calculated separately, did not perform as well.

Students generally did well on part (b).

Students who did well on part (a) also did so on part (c).

Most students had difficulty recognizing that the pond described in part (d) had cross sections perpendicular to the x-axis with area given by $(3 - x)\left(\sin(\pi x) - \left(x^3 - 4x\right)\right)$.

The mean score for this question was 4.89 for AB students and 6.38 for BC students out of a possible 9 points. About 9.8 percent of AB students and 24.1 percent of BC students earned all 9 points. About 7.6 percent of the AB students and 1.3 percent of BC students did not earn any points.

What were common student errors or omissions?

In part (a) many students partitioned the region R, which contributed to errors in determining the area of R. Students made errors when combining the areas of the individual parts of R. Some students did not include $\sin(\pi x)$ in the work leading to the area of R. Although this can produce a correct answer, some students did not explain why this upper boundary of the region could be excluded from the integrand.

In part (c) many students presented an incorrect integral that gave the volume of a solid of revolution, rather than the solid described in the problem.

In part (d) some students wrote the product of integrals rather than the integral of a product.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students need practice with determining the area of a region that crosses the x-axis.
- Students need practice with determining areas and volumes in contextual problems.
- Students should show their work and clearly show setups for integrals that are used in determining quantities such as area and volume.

Question AB2/BC2

What was the intent of this question?

This problem presented students with a table of data indicating the number of people $L(t)$ in line at a concert hall ticket office, sampled at seven times t during the 9 hours that tickets were being sold. (The question stated that $L(t)$ was twice differentiable.) Part (a) asked for an estimate for the rate of change of the number of people in line at a time that fell between the times sampled in the table. Students were to use data from the table to calculate an average rate of change to approximate this value. Part (b) asked for an estimate of the average number of people waiting in line during the first 4 hours and specified the use of a trapezoidal sum. Students needed to recognize that the computation of an average value involves a definite integral, approximate this integral with a trapezoidal sum, and then divide this total accumulation of people hours by 4 hours to obtain the average. Part (c) asked for the minimum number of solutions guaranteed for $L'(t) = 0$ during the 9 hours. Students were expected to recognize that a change in direction (increasing/decreasing) for a twice-differentiable function forces a value of 0 for its derivative. Part (d) provided the function $r(t) = 550te^{-t/2}$ tickets sold per hour as a model of the rate at which tickets were sold during the 9 hours and asked students to find the number of tickets sold in the first 3 hours, to the nearest whole number, using this model. Students needed to recognize that total tickets sold could be determined by a definite integral of the rate $r(t)$ at which tickets were sold.

How well did students perform on this question?

Many students did well in part (a). Parts (b) and (d) proved more difficult for some but were still accessible.

Students had difficulty earning all 3 points in part (c). To do so, they needed to have a complete argument that did not make unverifiable assumptions about the function L.

The mean score for this question was 3.36 for AB students and 5.10 for BC students out of a possible 9 points. About 0.6 percent of AB students and 1.8 percent of BC students earned all 9 points. About 21.6 percent of AB students and 5.8 percent of BC students did not earn any points.

What were common student errors or omissions?

Some students worked with the tabular function L as if its units were people per hour, rather than people. Common errors in part (a) included using an interval other than $[4, 7]$ and giving the number of people in line at $t = 5.5$ as the final answer.

In part (b) many students used the formula for the Trapezoidal Rule, even though it only applies when the subintervals are of equal length. Some students did not divide by the length of the interval to get the average value.

In part (c) the most common errors were: (1) assuming too much about the function L (e.g., that L was monotonic on the intervals defined by the *t*-values in the table); (2) not specifying clearly which function or interval the student was considering in a given statement; (3) making general statements about all points in an interval; and (4) not tying statements to data in the given table.

In part (d) the most common error was evaluating the rate function at 3, rather than integrating this function.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Teachers should offer students more practice with writing mathematical arguments. In writing these arguments, students should avoid using vague pronouns. They should also be careful to avoid references to “the function” when more than one function is under consideration.
- Students need practice working with functions presented in tabular form. Such functions present particular challenges in that precise behavior between specified points cannot be completely determined.
- Students need to be encouraged to include intermediate steps in their solutions.

Question AB3

What was the intent of this question?

This problem presented students with a scenario in which oil leaking from a pipeline into a lake organizes itself as a dynamic cylinder whose height and radius change with time. The rate at which oil is leaking into the lake was given as 2000 cubic centimeters per minute. Part (a) was a related-rates problem; students needed to use the chain rule to differentiate volume, $V = \pi r^2 h,$ with respect to time and determine the rate of change of the oil slick's height at an instant when the oil slick has radius 100 cm and height 0.5 cm, and its radius is increasing at $2.5 \text{ cm/min}.$ In part (b) an oil recovery device arrives on the scene; as the pipeline continues to leak at 2000 cubic centimeters per minute, the device removes oil at the rate of $R(t) = 400\sqrt{t}$ cubic centimeters per minute, with t measured in minutes from the time the device began removing oil. Students were asked for the time t when the volume of the oil cylinder is greatest. They needed to recognize the rate of change of the volume of oil in the lake, $\frac{dV}{dt},$ as the difference between the rate with which oil enters the lake from the leak and the rate at which it is removed by the device. A sign analysis of $\frac{dV}{dt}$ or an application of the Second Derivative Test and the critical point theorem could justify that the critical point found yields a maximum value for the volume of the oil cylinder. Part (c) tested students' ability to use the Fundamental Theorem of Calculus to find the amount of oil in the lake at the time found in part (b), given that 60,000 cubic centimeters had already leaked when the recovery device began its task.

How well did students perform on this question?

Many students correctly applied the product rule and chain rule and did quite well on part (a).

In part (b) many students had difficulty obtaining a correct expression for the rate of change of volume of the oil slick. Among those who could do this successfully, some had difficulty determining when the rate of change was zero, although most students recognized that this is what they needed to do in order to respond to the problem. Few students provided a complete and correct justification that the time they found was an absolute maximum.

Students performed fairly well on part (c).

The mean score for this question was 2.45 out of a possible 9 points. Only 0.3 percent of students earned all 9 points. About 28.1 percent of students did not earn any points.

What were common student errors or omissions?

In part (a) many students had difficulty using the product rule correctly to find $V'(t).$ In calculating the numerical answer, some students multiplied by π instead of dividing by it or made arithmetic errors in squaring.

In part (b) some students incorrectly interpreted the meaning of the quantities 2000 and $R(t)$, leading to errors in their setups. Many students did not correctly solve for the time t at which the rate of removal equaled the rate of leakage. Many students who did solve correctly did not provide sufficient justification for their answer.

Some students did not correctly use the initial condition in part (c).

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students need practice with using the product rule, quotient rule, and chain rule in a variety of settings.
- Students must know when to use a local argument and when to use a global argument in justifying extreme values.
- Students need practice with communicating mathematics through the use of clear, unambiguous, and mathematically precise language.

Question AB4/BC4

What was the intent of this question?

This problem presented students with the graph of a velocity function for a particle in motion along the *x*-axis for $0 \le t \le 6$. Areas of regions between the velocity curve and the *t*-axis were also given. Part (a) asked for the time and position of the particle when it is farthest left, so students needed to know that velocity is the derivative of position, and they had to be able to determine positions at critical times from the particle's initial position and areas of regions bounded by the velocity curve and the *t*-axis. Part (b) tested knowledge of the Intermediate Value Theorem applied to information about the particle's position function derived from its initial position and the supplied graph of its derivative. Part (c) asked students to interpret information about the speed of the particle from the velocity graph: namely, that if velocity is negative but increasing, then its absolute value, speed, is decreasing. Part (d) asked for the time intervals over which acceleration is negative, so students had to recognize that acceleration is the derivative of velocity. The sign of acceleration can be read from the intervals of increase/decrease of the velocity function.

How well did students perform on this question?

Student performance varied widely on this problem. Students who used the initial position correctly in part (a) and part (b) tended to do well on those parts. Many students who did well on parts (a) and (b) did not do well on parts (c) and (d).

The mean score for this question was 2.60 for AB students and 4.26 for BC students out of a possible 9 points. About 2.3 percent of AB students and 6.9 percent of BC students earned all 9 points. About 29.7 percent of AB students and 10.9 percent of BC students did not earn any points.

What were common student errors or omissions?

In part (a) many students did not correctly consider the entire time interval. Many students did not use the Extreme Value Theorem in their work. Some students tried to justify their answers using the First Derivative Test. Some students used an incorrect position for the particle at time $t = 0$.

In part (b) many students did not consider the region from $t = 5$ to $t = 6$.

In part (c) some students stated that the speed was increasing. Many students who did find that the speed was decreasing did not provide a complete and correct reason for their answer.

In part (d) many students included $t = 1$ and $t = 4$ in their answers, but these are places where the acceleration was zero.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students need practice with recognizing when a function is monotonically increasing or decreasing. This is especially important in contexts such as particle motion.
- Students need practice with problems that require use of the Intermediate Value Theorem.
- Students need practice communicating mathematics through the use of clear, unambiguous, and mathematically precise language.
- Students must label graphs and diagrams with all of the appropriate information.

Question AB5

What was the intent of this question?

This problem presented students with a separable differential equation. In part (a) they were asked to sketch its slope field at nine sample points. Part (b) asked for the solution to the differential equation with a given initial condition. The solution involved selection of the portion of $\ln|y-1|$ that includes the initial condition. Part (c) asked for the limit of the solution from part (b) as $x \to \infty$.

How well did students perform on this question?

Most students were able to earn at least 1 of 2 points in part (a). Most students were able to earn some points in part (b). Some students did not arrive at an answer in part (b) and therefore did not continue on to part (c).

The mean score for this question was 3.70 out of a possible 9 points. About 1.6 percent of students earned all 9 points. About 15.1 percent of students did not earn any points.

What were common student errors or omissions?

In part (a) some students placed slope segments on the y-axis.

In part (b) some students did not attempt to separate variables or did not separate variables correctly. Some students who correctly separated variables omitted the absolute value signs in the antiderivative with respect to y or removed the absolute values at a point that resulted in a statement that is not true for the initial condition.

Many students did not know how to answer part (c).

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Teachers should continue to emphasize properties of logarithmic functions, particularly their domains and ranges.
- Students need practice with calculating limits at infinity.
- Students need to be encouraged to include intermediate steps in their solutions.

Question AB6

What was the intent of this question?

This problem presented students with a function f defined by $f(x) = \dfrac{\ln x}{x}$ for $x > 0,$ together with a formula for $f'(x).$ Part (a) asked for an equation of the line tangent to the graph of f at $x = e^2.$ In part (b) students needed to solve $f'(x) = 0$ and determine the character of this critical point from the supplied $f'(x).$ In part (c) students had to demonstrate skill with the quotient rule to obtain a formula for $f''(x)$ and solve $f''(x) = 0$ to find the x-coordinate of what was promised to be the only point of inflection for the graph of f. Part (d) tested students' knowledge of properties of $\ln x$ to determine the limit of $f(x)$ as $x \to 0^+$.

How well did students perform on this question?

Student performance on this question was average. In parts (a), (b), and (c) most students were able to enter the problem, but they made algebraic simplification errors. Many students were able to use the quotient rule to find the second derivative in part (c). Most students had difficulty determining the limit in part (d).

The mean score for this question was 3.06 out of a possible 9 points. About 2.1 percent of students earned all 9 points. About 28.9 percent of students did not earn any points.

What were common student errors or omissions?

In part (a) many students made errors in simplifying expressions involving e and $\ln$.

In part (b) most students did not present a complete and correct justification. Some presented a sign chart or table with values of f'. Without verbal commentary explaining the chart or table, this was insufficient justification.

In part (c) many students who were able to correctly use the quotient rule to find $f''(x)$ then made an error in simplifying the expression.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Teachers may prefer that their students simplify arithmetic in their classrooms, but arithmetic expressions need not be simplified on the AP Calculus Exams.
- Students need practice with writing justifications. Sign charts and other charts showing function behavior are by themselves incomplete justifications.
- Students must show the work that leads to their answers.

Question BC3

What was the intent of this question?

This problem presented students with a table of values for a function h and its derivatives up to the fourth order at $x = 1,$ $x = 2,$ and $x = 3.$ The question stated that h has derivatives of all orders, and that the first four derivatives are increasing on $1 \le x \le 3.$ Part (a) asked for the first-degree Taylor polynomial about $x = 2$ and the approximation for $h(1.9)$ given by this polynomial. Students needed to use the given information to determine that the graph of h is concave up between $x = 1.9$ and $x = 2$ to conclude that this approximation is less than the value of $h(1.9)$. Part (b) asked for the third-degree Taylor polynomial about $x = 2$ and the approximation for $h(1.9)$ given by this polynomial. In part (c) students were expected to observe that the given conditions imply that $\left|h^{(4)}(x)\right|$ is bounded above by $h^{(4)}(2)$ on $1.9 \le x \le 2$ and apply this to the Lagrange error bound to show that the estimate in part (b) has error less than 3×10^{-4}.

How well did students perform on this question?

Students performed very well on this problem. This is encouraging given the fact that problems involving series are often challenging for students.

In part (a) most students successfully found the first-degree Taylor polynomial. Most students correctly stated that the estimate of $h(1.9)$ was an underestimate, but many did not provide a sufficient reason.

In parts (b) and (c) most students earned some, but not all, points.

The mean score for this question was 4.42 out of a possible 9 points. Only 0.5 percent of students earned all 9 points. About 15.4 percent of students did not earn any points.

What were common student errors or omissions?

In part (a) many students did not provide an appropriate reason to support their claim that the approximation is less than the actual value of the function at 1.9. Also, many students presented polynomials in part (a) that contained terms beyond the linear terms.

In part (b) some students made errors in reading values from the table.

In part (c) many students did not appropriately use the Lagrange error bound. Students needed to make reference to the maximum value of the fourth derivative of f on the appropriate interval in order to earn all of the points in part (c).

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students should not equate an approximation with the curve being approximated.
- Students need practice with writing justifications. Students should clearly state the connections between information given in the problem and the conclusions they are drawing in their answers.
- Students need more practice with using various error bounds.

Question BC5

What was the intent of this question?

In this problem, students were told that a function f has derivative $f'(x) = (x - 3)e^x$ and that $f(1) = 7$. In part (a) students needed to determine with justification the character of the critical point for f at $x = 3$. Part (b) asked for the intervals on which the graph of f is both decreasing and concave up. For this, students had to apply the product rule to obtain $f''(x)$. In part (c) students needed to solve the initial value problem to find $f(3)$, employing integration by parts along the way.

How well did students perform on this question?

Students performed well on this problem. Some students did not correctly justify or explain their answers in parts (a) and (b). In part (c), once students realized that they must perform integration by parts, they did very well.

The mean score for this question was 5.82 out of a possible 9 points. About 12.3 percent of students earned all 9 points. Only 4.4 percent of students did not earn any points.

What were common student errors or omissions?

In part (a) some students tried to justify by using the fact that f changed from decreasing to increasing, instead of justifying by making reference to the behavior of f' or of f''.

In part (b) some students did not give a correct explanation of how they arrived at their answer.

In part (c) some students did not use integration by parts.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students need practice with writing justifications. Sign charts and other charts showing function behavior are by themselves incomplete justifications.
- Students must show the work that leads to their answers.

Question BC6

What was the intent of this question?

This problem presented students with a logistic differential equation and the initial value $f(0) = 8$ of a particular solution $y = f(t)$. In part (a) a slope field for the differential equation was given, and students were asked to sketch solution curves through two specified points. In particular, students should have demonstrated appropriate behavior for these curves for $t \geq 0$, especially with regard to the horizontal lines $y = 0$ and $y = 6$. For part (b) students needed to use the given initial value for the solution f and a two-step Euler's method to approximate $f(1)$. In part (c) students were directed to find the second-degree Taylor polynomial for f about $t = 0$ and use it to approximate $f(1)$. Part (d) asked for the range of the particular solution $y = f(t)$.

How well did students perform on this question?

Overall, students did very well on parts (a) and (b), with many students earning all of the points possible. In part (c) many students earned 1 or fewer points. In part (d) most students failed to indicate the correct interval.

The mean score for this question was 3.84 out of a possible 9 points. About 1.9 percent of students earned all 9 points. Only 3.6 percent of students did not earn any points.

What were common student errors or omissions?

In part (a) some students drew solutions that crossed the asymptotes at $y = 0$ and/or $y = 6$.

In part (b) some students made arithmetic errors. Some students used a tabular approach to Euler's method but failed to clarify the individual steps. The use of the tabular method became especially difficult to evaluate when the student made an error. In such cases, it was difficult or impossible to award partial credit.

In part (c) many students did not use the chain rule to find $\frac{d^2y}{dt^2}$. Many students used $y = 0$ instead of $y = 8$ when evaluating $\frac{dy}{dt}$ at $t = 0$ or $\frac{d^2y}{dt^2}$ at $t = 0$.

In part (d) many students used incorrect notation or did not present the correct interval.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Teachers should emphasize the need to draw curves in slope fields that illustrate all of the important behaviors of the solutions, including asymptotic behavior.
- Although the tabular approach to Euler's method can provide a useful framework for a student to work through the steps of Euler's method, students who use this approach must also be able to show how they obtained the values they entered into the table. Students should be encouraged to show their work in calculating values in the table regardless of the relative difficulty of the calculations.
- Students should be encouraged to proofread carefully and to check their arithmetic and algebraic computations.