AP® CALCULUS AB
2009 SCORING GUIDELINES (Form B)

Question 6

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	–10	–8	–4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

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1 : units in (a) and (b)

(a) $a(36) = v'(36) \approx \dfrac{v(40) - v(32)}{40 - 32} = \dfrac{11}{8}$ meters/sec^2

1 : answer

(b) $\int_{20}^{40} v(t)\,dt$ is the particle's change in position in meters from time $t = 20$ seconds to time $t = 40$ seconds.

$$\int_{20}^{40} v(t)\,dt \approx \frac{v(20) + v(25)}{2} \cdot 5 + \frac{v(25) + v(32)}{2} \cdot 7 + \frac{v(32) + v(40)}{2} \cdot 8$$
$$= -75 \text{ meters}$$

3 : { 1 : meaning of $\int_{20}^{40} v(t)\,dt$; 2 : trapezoidal approximation }

(c) $v(8) > 0$ and $v(20) < 0$
$v(32) < 0$ and $v(40) > 0$
Therefore, the particle changes direction in the intervals $8 < t < 20$ and $32 < t < 40$.

2 : { 1 : answer; 1 : explanation }

(d) Since $v'(t) = a(t) > 0$ for $0 < t < 8$, $v(t) \ge 3$ on this interval.
Therefore, $x(8) = x(0) + \int_0^8 v(t)\,dt \ge 7 + 8 \cdot 3 > 30$.

2 : { 1 : $v'(t) = a(t)$; 1 : explanation of $x(8) > 30$ }

NO CALCULATOR ALLOWED

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

Work for problem 6(a)

$a(36) \approx \frac{v(40)-v(32)}{40-32}$

$= \frac{7-(-4)}{8} = \frac{11}{8}$ m/s^2

Work for problem 6(b)

$\int_{20}^{40} v(t)\,dt \approx 5\left(\frac{-8+(-10)}{2}\right) + 7\left(\frac{-4+(-8)}{2}\right) + 8\left(\frac{7+(-4)}{2}\right)$

$= 5(-9) + 7(-6) + 8\left(\frac{3}{2}\right)$

$= -45 - 42 + 12 = -75$ meters

total, not net

$\int_{20}^{40} v(t)\,dt$ is the total displacement of the particle from $t = 20$ seconds to $t = 40$ seconds

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NO CALCULATOR ALLOWED

Work for problem 6(c)

Since v(t) is differentiable, v(t) is continuous.

Particle changes direction → v(t) changes sign.

The particle must change direction in (8, 20) and in (32, 40).

$v(8) = 5 > 0$
$v(20) = -10 < 0$
} v(t) changes sign at some t for $8 < t < 20$ (v(t) = 0)

$v(32) = -4 < 0$
$v(40) = 7 > 0$
} v(t) changes sign at some d for $32 < d < 40$

The above is true due do Intermediate Value Theorem.
Since v(t) changes sign in (8, 20) and in (32, 40),
the particle must change direction in (8,20) and in (32,40)

Work for problem 6(d)

$a(t) > 0$ for $0 < t < 8$ seconds.

Thus, v(t) is increasing for $0 < t < 8$ seconds.

Since $v(0) = 3$ m/s, and $v(t) > 0$ on $0 < t < 8$,

absolute minimum of v(t) on $0 < t < 8$ is 3 m/s.

At 3 m/s, minimum distance travelled from $t = 0$ to $t = 8$

is $\int_0^8 v(t)\,dt = \int_0^8 3\,dt = 3 \times 8 = 24$ metres.

$x(8) = x(0) + \int_0^8 v(t)\,dt = 7 + \int_0^8 v(t)\,dt$.

Since, $\int_0^8 v(t)\,dt \geq 24$ metres, $x(8) \geq 31$ metres and 31 > 30

Thus, position of particle at $t = 8$ seconds must be greater than $x = 30$ metres

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NO CALCULATOR ALLOWED

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Work for problem 6(a)

$a(36) = \dfrac{V(40) - V(32)}{40 - 32} = \dfrac{11}{8}$ meters per second square

Work for problem 6(b)

$\dfrac{L(20) + L(25)}{2}(25-20) + \dfrac{L(25) + L(32)}{2}(32-25) + \dfrac{L(40) + L(32)}{2}(40-32)$

$= \dfrac{-10-8}{2} \cdot 5 + \dfrac{-8-4}{2} \cdot 7 + \dfrac{-4+7}{2} \cdot 8 = -75$ m

This is the distance that the particle traveled during $20 < t < 40$, which is 75 m left.

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NO CALCULATOR ALLOWED

Work for problem 6(c)

Yes. In $8 < t < 10$ and $32 < t < 40$

Because the velocity changes from positive to negative during those subintervals.

Work for problem 6(d)

Because the acceleration of the particle is positive for $0 < t < 8$, so the velocity of the particle must be increasing from $t = 0$ to $t = 8$, from 3 m/s to 5 m/s.

Suppose the velocity is 3 m/s, after 8 seconds. The particle will travel 24 meters.

24 meters plus the initial 7 meters is 31 meters.

So, by using the slowest of velocity, the car still can travel more than 30 meters.

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NO CALCULATOR ALLOWED

t (seconds)	0	8	20	25	32	40
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Work for problem 6(a)

By the Mean Value Theorem:

$$a(36) = \frac{v(40) - v(32)}{40 - 32} = \frac{11}{8} \text{ (meters/seconds}^2\text{)}$$

Work for problem 6(b)

$\int_{20}^{40} v(t)\,dt$ shows us the overall sum of changes of $v(t)$ during 20 seconds from $t = 20$ to $t = 40$.

$$\int_{20}^{40} v(t)\,dt \approx (9 \cdot 5 + 6 \cdot 7 + 8 \cdot 9) = 45 + 42 + 72 = 159$$

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Work for problem 6(c)

Yes, It must change the direction on the given intervals $t \in (8; 20)$ and $t \in (32; 40)$, because velocity changes its sign on these intervals

Work for problem 6(d)

As $V'(t) = a(t)$, $a(t)$ is positive for $0 < t < 8$ seconds, $V(t)$ is also positive, therefore, $V(t)$ is increasing for $0 < t < 8$ seconds;

$X'(t) = V(t)$

$X(8) = X(0) + \int_0^8 (V(t) \cdot dt)$; $X(0) = 7$, $\int_0^8 V(t) \cdot dt$ is more than 23 (32, for instance, using trapezoidal rule),

so, $X(8) > 30$ meters

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AP® CALCULUS AB
2009 SCORING COMMENTARY (Form B)

Question 6

Sample: 6A
Score: 9

The student earned all 9 points. Note that in part (b) students could include units in either the numerical answer or the verbal description. The student's use of "total" is not necessary.

Sample: 6B
Score: 6

The student earned 6 points: the units point, 1 point in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student's answer is correct. The use of an equality sign instead of an approximation symbol was ignored. In part (b) the student did not earn the point for the meaning of the definite integral, because the response uses "distance" instead of net distance. The student earned 2 points for the trapezoidal approximation; the use of L instead of v was ignored. In part (c) the student has only one correct interval, and the justification is inconsistent with that correct interval. The student was eligible for a point only if the justification matched the correct interval. In part (d) the student's work is correct. The verbal argument notes that the velocity is increasing, implies that $v(t) \geq 3$ on the interval, and argues from the initial position plus distance traveled.

Sample: 6C
Score: 4

The student earned 4 points: no units point, 1 point in part (a), no points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student does not include units and is not using a trapezoidal approximation. In part (c) the student's work is correct. The student was not required to describe the nature of the sign changes in $v(t)$. In part (d) the student earned the first point. There is no valid explanation as to why the definite integral is more than 23. The student needs to appeal to the fact that $v(t) \geq 3$ for $0 < t < 8$.