CollegeBoard
Advanced Placement Program

# AP® Calculus AB
# 2009 Free-Response Questions

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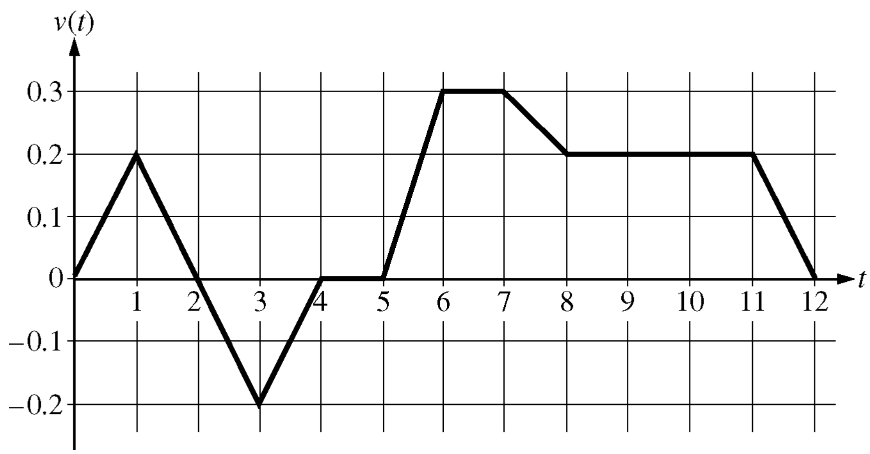
**CALCULUS AB**
**SECTION II, Part A**
**Time—45 minutes**
**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**




1. Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

   (a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

   (c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

   (d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function $w$ given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

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2. The rate at which people enter an auditorium for a rock concert is modeled by the function $R$ given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

   (a) How many people are in the auditorium when the concert begins?

   (b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

   (c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function $w$ models the total wait time for all the people who enter the auditorium before time $t$. The derivative of $w$ is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

   (d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

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3. Mighty Cable Company manufactures cable that sells for $120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is $x$ meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

   (a) Find Mighty's profit on the sale of a 25-meter cable.

   (b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\, dx$ in the context of this problem.

   (c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is $k$ meters long.

   (d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

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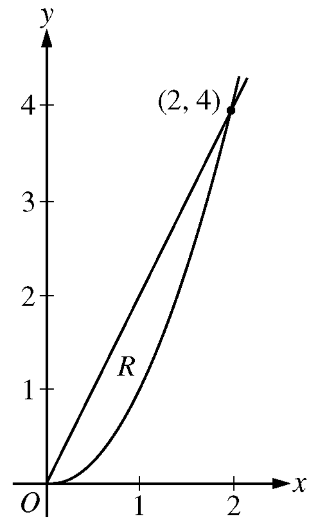
**END OF PART A OF SECTION II**

**CALCULUS AB**
**SECTION II, Part B**
**Time—45 minutes**
**Number of problems—3**

**No calculator is allowed for these problems.**




4. Let $R$ be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.
   (a) Find the area of $R$.
   (b) The region $R$ is the base of a solid. For this solid, at each $x$ the cross section perpendicular to the $x$-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
   (c) Another solid has the same base $R$. For this solid, the cross sections perpendicular to the $y$-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

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| $x$ | 2 | 3 | 5 | 8 | 13 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 4 | –2 | 3 | 6 |

5. Let $f$ be a function that is twice differentiable for all real numbers. The table above gives values of $f$ for selected points in the closed interval $2 \le x \le 13$.

   (a) Estimate $f'(4)$. Show the work that leads to your answer.

   (b) Evaluate $\int_2^{13} (3 - 5f'(x))\, dx$. Show the work that leads to your answer.

   (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\, dx$. Show the work that leads to your answer.

   (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all $x$ in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of $f$ at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of $f$ on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

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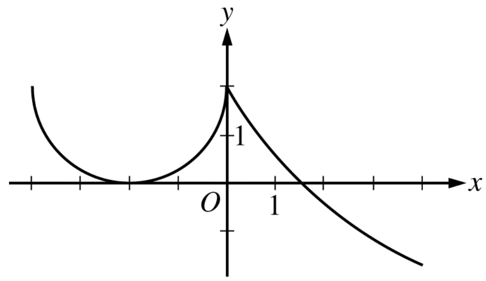


Graph of $f'$

6. The derivative of a function $f$ is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3} - 3 & \text{for } \quad 0 < x \le 4 \end{cases}$.

   The graph of the continuous function $f'$, shown in the figure above, has $x$-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of $g$ on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.

   (a) For $-4 < x < 4$, find all values of $x$ at which the graph of $f$ has a point of inflection. Justify your answer.

   (b) Find $f(-4)$ and $f(4)$.

   (c) For $-4 \le x \le 4$, find the value of $x$ at which $f$ has an absolute maximum. Justify your answer.

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**END OF EXAM**