



**AP<sup>®</sup> Calculus AB**  
**2010 Free-Response Questions**  
**Form B**

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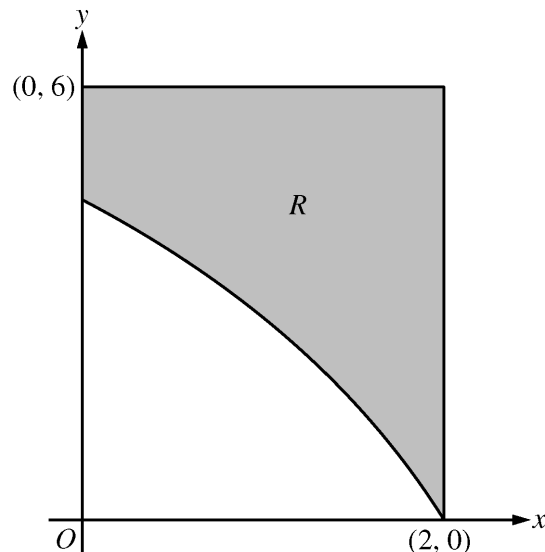
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**CALCULUS AB**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

A graphing calculator is required for some problems or parts of problems.



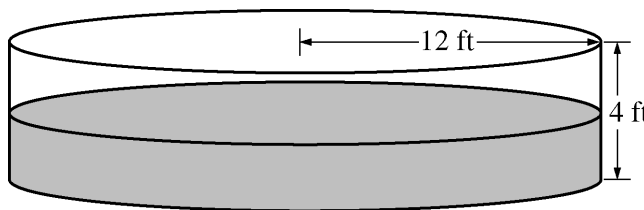
1. In the figure above,  $R$  is the shaded region in the first quadrant bounded by the graph of  $y = 4\ln(3 - x)$ , the horizontal line  $y = 6$ , and the vertical line  $x = 2$ .
  - (a) Find the area of  $R$ .
  - (b) Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 8$ .
  - (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of the solid.

**WRITE ALL WORK IN THE EXAM BOOKLET.**

2010 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

2. The function  $g$  is defined for  $x > 0$  with  $g(1) = 2$ ,  $g'(x) = \sin\left(x + \frac{1}{x}\right)$ , and  $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$ .
- Find all values of  $x$  in the interval  $0.12 \leq x \leq 1$  at which the graph of  $g$  has a horizontal tangent line.
  - On what subintervals of  $(0.12, 1)$ , if any, is the graph of  $g$  concave down? Justify your answer.
  - Write an equation for the line tangent to the graph of  $g$  at  $x = 0.3$ .
  - Does the line tangent to the graph of  $g$  at  $x = 0.3$  lie above or below the graph of  $g$  for  $0.3 < x < 1$ ? Why?

$t$	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



3. The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time  $t = 0$ . During the time interval  $0 \leq t \leq 12$  hours, water is pumped into the pool at the rate  $P(t)$  cubic feet per hour. The table above gives values of  $P(t)$  for selected values of  $t$ . During the same time interval, water is leaking from the pool at the rate  $R(t)$  cubic feet per hour, where  $R(t) = 25e^{-0.05t}$ . (Note: The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is given by  $V = \pi r^2 h$ .)
- Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval  $0 \leq t \leq 12$  hours. Show the computations that lead to your answer.
  - Calculate the total amount of water that leaked out of the pool during the time interval  $0 \leq t \leq 12$  hours.
  - Use the results from parts (a) and (b) to approximate the volume of water in the pool at time  $t = 12$  hours. Round your answer to the nearest cubic foot.
  - Find the rate at which the volume of water in the pool is increasing at time  $t = 8$  hours. How fast is the water level in the pool rising at  $t = 8$  hours? Indicate units of measure in both answers.

WRITE ALL WORK IN THE EXAM BOOKLET.

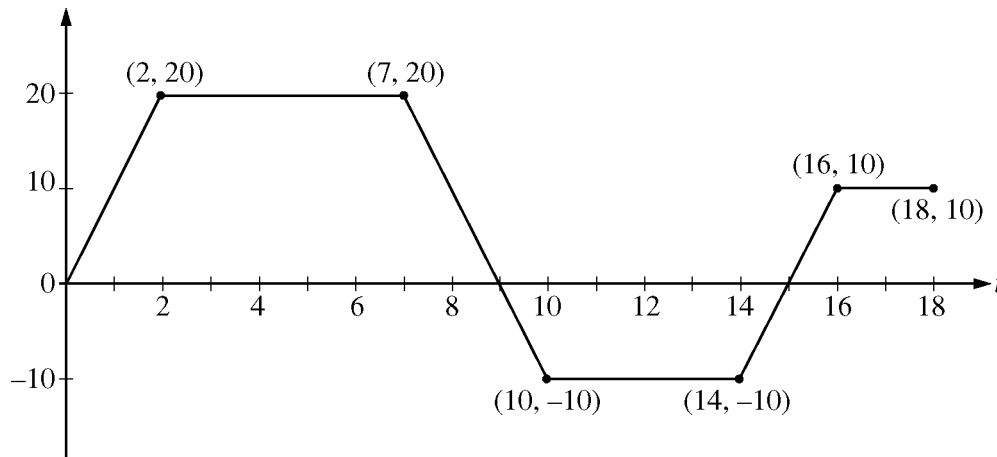
END OF PART A OF SECTION II

CALCULUS AB  
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of  $v$

4. A squirrel starts at building  $A$  at time  $t = 0$  and travels along a straight, horizontal wire connected to building  $B$ . For  $0 \leq t \leq 18$ , the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.
- At what times in the interval  $0 < t < 18$ , if any, does the squirrel change direction? Give a reason for your answer.
  - At what time in the interval  $0 \leq t \leq 18$  is the squirrel farthest from building  $A$ ? How far from building  $A$  is the squirrel at that time?
  - Find the total distance the squirrel travels during the time interval  $0 \leq t \leq 18$ .
  - Write expressions for the squirrel's acceleration  $a(t)$ , velocity  $v(t)$ , and distance  $x(t)$  from building  $A$  that are valid for the time interval  $7 < t < 10$ .

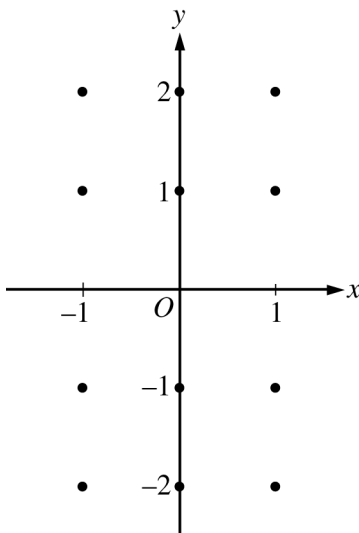
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5. Consider the differential equation  $\frac{dy}{dx} = \frac{x+1}{y}$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for  $-1 < x < 1$ , sketch the solution curve that passes through the point  $(0, -1)$ .

(Note: Use the axes provided in the exam booklet.)



- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the  $xy$ -plane for which  $y \neq 0$ . Describe all points in the  $xy$ -plane,  $y \neq 0$ , for which  $\frac{dy}{dx} = -1$ .
- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = -2$ .

6. Two particles move along the  $x$ -axis. For  $0 \leq t \leq 6$ , the position of particle  $P$  at time  $t$  is given by

$$p(t) = 2 \cos\left(\frac{\pi}{4}t\right), \text{ while the position of particle } R \text{ at time } t \text{ is given by } r(t) = t^3 - 6t^2 + 9t + 3.$$

- (a) For  $0 \leq t \leq 6$ , find all times  $t$  during which particle  $R$  is moving to the right.
- (b) For  $0 \leq t \leq 6$ , find all times  $t$  during which the two particles travel in opposite directions.
- (c) Find the acceleration of particle  $P$  at time  $t = 3$ . Is particle  $P$  speeding up, slowing down, or doing neither at time  $t = 3$ ? Explain your reasoning.
- (d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval  $1 \leq t \leq 3$ .

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM