Question 1

In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.
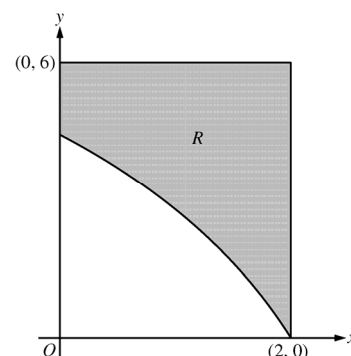
(a) $\displaystyle\int_0^2 (6 - 4\ln(3 - x))\,dx = 6.816$ or 6.817

1 : Correct limits in an integral in (a), (b), or (c)

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\displaystyle\pi\int_0^2 \left((8 - 4\ln(3 - x))^2 - (8 - 6)^2\right) dx$

$= 168.179$ or 168.180

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\displaystyle\int_0^2 (6 - 4\ln(3 - x))^2\, dx = 26.266$ or 26.267

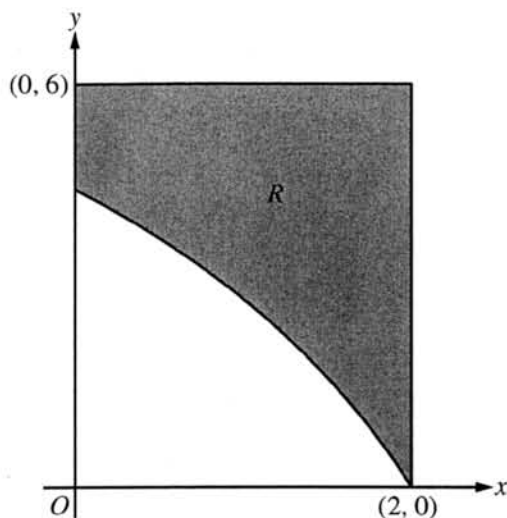$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

$(0, 6)$

R

O

$(2, 0)$

Work for problem 1(a)

$$S_R = 2 \times 6 - \int_0^2 4\ln(3-x)\,dx$$

$$= 12 - (5.183) \approx 6.817$$

Do not write beyond this border.

-4-

Work for problem 1(b)

assum $y_1 = 4\ln(3-x)$

$\qquad y_2 = 6$

$V = \pi\int_0^2 \left[\dfrac{(8-y_1)^2}{?} - \dfrac{(8-y_2)^2}{?}\right]dx$

$\quad = \pi\int_0^2 \left[(8-y_1)^2 - (8-y_2)^2\right]dx$

$\quad = \pi\int_0^2 \left[(8-4\ln(3-x))^2 - (8-6)^2\right]dx$

$\quad \approx 168.180.$

Work for problem 1(c)

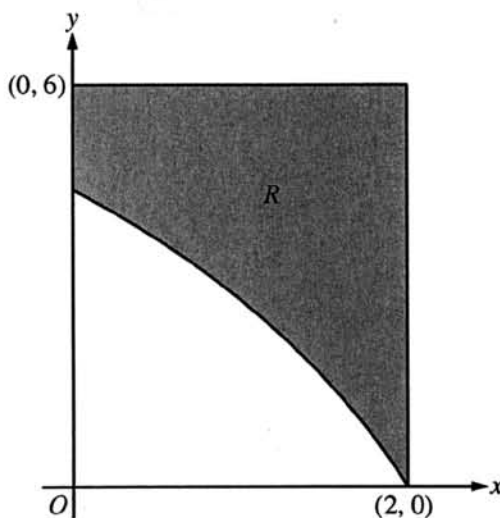$V = \int_0^2 \left[6 - 4\ln(3-x)\right]^2 dx.$

$\quad \approx 26.267.$

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$R = \int_0^2 6 - 4\ln(3-x)\, dx$$

$$= \int_0^2 6\, dx - \int_0^2 4\ln(3-x)\, dx$$

$$= 6x \Big/_0^2 - 4\int_0^2 \ln(3-x)\, dx$$

$$f = \ln(3-x) \quad f' = \frac{-1}{3-x}$$
$$g' = 1 \quad g = x$$

$$= 12 - \left(4x\ln(3-x)\Big/_0^2 - \int_0^2 \frac{-x}{3-x}\, dx\right)$$

$$= 6.817 \text{ units}^2$$

-4-

Work for problem 1(b)

$$V = \pi \int_0^2 [8 - 4 \cdot \ln(3-x)]^2 - (8-6)^2 \, dx$$

$$= \pi \int_0^2 \left([8 - 4\ln(3-x)]^2 - 2^2\right) dx$$

$$= 168.180 \text{ units}^3$$

Work for problem 1(c)

$$V = \int_0^2 (2 \times 6)^2 - (2 \times 4\ln(3-x))^2 \, dy$$

$$= \int_0^2 12^2 - (8\ln(3-x))^2 \, dx$$
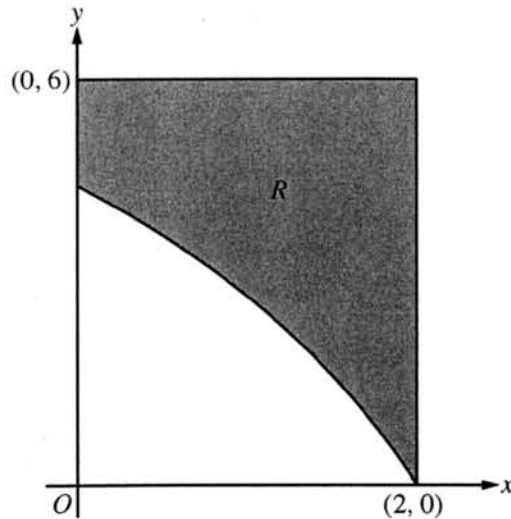
$$= 222.133 \text{ units}^3$$

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$A = \int_0^2 (6 - 4\ln(3-x))\, dx$$

with the help of the calculator, we type this in

and we get $A = \int_0^2 (6 - 4\ln(3-x))\, dx = 6.81~\cancel{766668}$

-4-

Do not write beyond this border.

Work for problem 1(b)

Since ~~it R~~ is revolved about $\ell: y=8$
we use the washer method

$$V = \pi \int_0^2 (8 - 6^2)(8 - (4\ln(3-x))^2) \, dx$$

with the calculator, we get

$$V = \pi \int_0^2 (8 - 6^2)(8 - (4\ln(3-x))^2) \, dx = 41.059$$

Work for problem 1(c)

The side of the square shall be $y = 4\ln(3-x)$
which makes the Area of the square as $A = (4\ln(3-x))^2$

$$V = \pi \int_0^2 (4\ln(3-x))^2 \, dx$$

We use the calculator to get

$$V = \pi \int_0^2 (4\ln(3-x))^2 \, dx = 51.732$$

Question 1

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: the global limits point, 2 points in part (a), 3 points in part (b), and no points in part (c). In part (a) the student earned both points and the global limits point. The student's intermediate work includes a misplaced 4, but the correct numerical answer was treated as a restart since this was the calculator portion of the exam. In part (b) the student's work is correct. In part (c) the student does not use square cross sections and was not eligible for any points.

Sample: 1C
Score: 3

The student earned 3 points: the global limits point, 2 points in part (a), no points in part (b), and no points in part (c). In part (a) the student earned the global limits point and has correct work. In part (b) the student attempts to find the volume using washers, but the work is incorrect. In part (c) the student uses an incorrect width for the area of the square cross section and includes a factor of π.