AP® CALCULUS AB
2010 SCORING GUIDELINES (Form B)

Question 2

The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

(a) Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.

(b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 0.3$.

(d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

(a) The graph of g has a horizontal tangent line when $g'(x) = 0$.
This occurs at $x = 0.163$ and $x = 0.359$.

$2 : \begin{cases} 1 : \text{sets } g'(x) = 0 \\ 1 : \text{answer} \end{cases}$

(b) $g''(x) = 0$ at $x = 0.129458$ and $x = 0.222734$

The graph of g is concave down on $(0.1295, 0.2227)$ because $g''(x) < 0$ on this interval.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) $g'(0.3) = -0.472161$

$g(0.3) = 2 + \int_1^{0.3} g'(x)\,dx = 1.546007$

An equation for the line tangent to the graph of g is $y = 1.546 - 0.472(x - 0.3)$.

$4 : \begin{cases} 1 : g'(0.3) \\ 1 : \text{integral expression} \\ 1 : g(0.3) \\ 1 : \text{equation} \end{cases}$

(d) $g''(x) > 0$ for $0.3 < x < 1$

Therefore the line tangent to the graph of g at $x = 0.3$ lies below the graph of g for $0.3 < x < 1$.

1 : answer with reason

Work for problem 2(a)

Horizontal tangent line : $g'(x) = 0$

$\therefore \sin\left(x + \frac{1}{x}\right) = 0$

$x = 0.163$ & 0.359

There exists horizontal tangent lines at $x = 0.163$ and $x = 0.359$ *

Work for problem 2(b)

$g''(x) < 0$, g concaved down.

$\therefore g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right) < 0$

$g''(x) < 0$ on $(0.129, 0.223)$

Continue problem 2 on page 7.

Work for problem 2(c)

$g'(0.3) = -0.472$

$\int_{0.3}^{1} \sin(x+\frac{1}{x})\,dx = 0.45399$

$g(0.3) = 2 + (-0.45399)$

$= 1.546$

$y - 1.546 = -0.472(x - 0.3)$

$y = -0.472x + 1.688$ *

Do not write beyond this border.

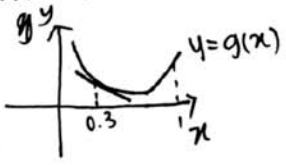
Work for problem 2(d)

At $x = 0.3$

$g''(0.3) = 8.913 > 0$ $\quad g'(0.3) = -0.472 < 0$

∴ g is concaved up on $x = 0.3$ * g is concaved up on (0.3, 1) as g''(x) remains positive on (0.3,1)

As such the tangent of graph g at x=0.3 will lie below the graph of g



GO ON TO THE NEXT PAGE.

Work for problem 2(a)

$g'(x) = 0$

$0 = \sin\left(x + \frac{1}{x}\right)$

$x = 0.163$

$x = 0.359$

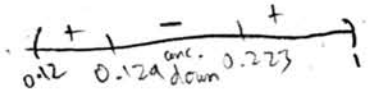
Work for problem 2(b)

g is concave down on the interval (0.129, 0.223) because g' is decreasing on this interval and $g'' < 0$ on this interval.

$g'' = 0$

$x = 0.129$

$x = 0.223$



Do not write beyond this border.

Continue problem 2 on page 7.

Work for problem 2(c)

$m = g'(0.3) = \sin\left(0.3 + \frac{1}{0.3}\right) = -0.472161$

$(1, 2) \quad y = mx + b$

$2 = (-0.472161)(1) + b$

$2.47216 = b$

$y = -0.472x + 2.472$

Do not write beyond this border.

Do not write beyond this border.

Work for problem 2(d)

The line tangent to g at $x = 0.3$ lies below the graph of g for $0.3 < x < 1$ because on the interval $0.3 < x < 1$ g is concave up.

Work for problem 2(a)

∵ It has horizontal tangent line

∴ $g'(x) = 0$

∴ $\sin(x+\frac{1}{x}) = 0$

∴ $x = 0.163$ or 0.359

∴ When x is equal to 0.163 or 0.359, the graph of g has a horizontal tangent

Work for problem 2(b)

∵ g is concave down

∴ $g'' < 0$

∴ $(1-\frac{1}{x^2})\cos(x+\frac{1}{x}) < 0$

∵ $(1-\frac{1}{x^2})\cos(x+\frac{1}{x})$ cannot be smaller than 0 in the domain $(0.12, 1)$

∴ There is no subinterval in $(0.12, 1)$ that the graph g is concave down

Continue problem 2 on page 7.

Work for problem 2(c)

$\because g'(x) = \sin(x+\frac{1}{x})$

$\therefore g(x) = \int \sin(x+\frac{1}{x})dx$

let $u = x+\frac{1}{x}$

$\therefore du =$

let line tangent is $y = ax+b$

$\therefore a = g'(x) = \sin(x+\frac{1}{x})$

$\because x = 0.3$

$\therefore a = g'(0.3) = -0.472$.

$\therefore y = -0.472x + b$

Work for problem 2(d)

$g'(0.3) = -0.472$

$g'(1) = 0.909$

Do not write beyond this border.

AP® CALCULUS AB
2010 SCORING COMMENTARY (Form B)

Question 2

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 1 point in part (d). In parts (a) and (b), the student's work is correct. In part (c) the student earned the slope point for $g'(0.3)$. In part (d) the student's work is correct.

Sample: 2C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student's concavity statement is incorrect. In part (c) the student earned the slope point for $g'(0.3)$. In part (d) the student does not include a statement about the tangent line.