



**AP® Calculus AB  
2010 Scoring Guidelines  
Form B**

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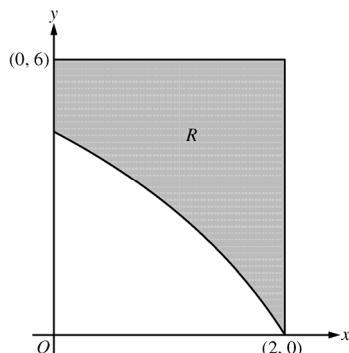
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**Question 1**

In the figure above,  $R$  is the shaded region in the first quadrant bounded by the graph of  $y = 4\ln(3 - x)$ , the horizontal line  $y = 6$ , and the vertical line  $x = 2$ .

- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 8$ .
- (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of the solid.



(a)  $\int_0^2 (6 - 4\ln(3 - x)) \, dx = 6.816 \text{ or } 6.817$

1 : Correct limits in an integral in (a), (b), or (c)

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b)  $\pi \int_0^2 ((8 - 4\ln(3 - x))^2 - (8 - 6)^2) \, dx$   
 $= 168.179 \text{ or } 168.180$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c)  $\int_0^2 (6 - 4\ln(3 - x))^2 \, dx = 26.266 \text{ or } 26.267$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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**Question 2**

The function  $g$  is defined for  $x > 0$  with  $g(1) = 2$ ,  $g'(x) = \sin\left(x + \frac{1}{x}\right)$ , and  $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$ .

- (a) Find all values of  $x$  in the interval  $0.12 \leq x \leq 1$  at which the graph of  $g$  has a horizontal tangent line.
- (b) On what subintervals of  $(0.12, 1)$ , if any, is the graph of  $g$  concave down? Justify your answer.
- (c) Write an equation for the line tangent to the graph of  $g$  at  $x = 0.3$ .
- (d) Does the line tangent to the graph of  $g$  at  $x = 0.3$  lie above or below the graph of  $g$  for  $0.3 < x < 1$ ? Why?

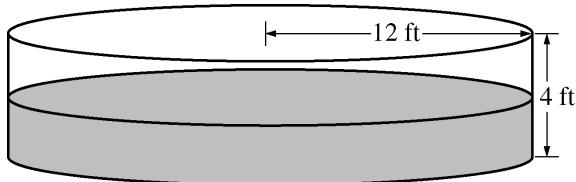
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<p>(a) The graph of <math>g</math> has a horizontal tangent line when <math>g'(x) = 0</math>. This occurs at <math>x = 0.163</math> and <math>x = 0.359</math>.</p> <p>(b) <math>g''(x) = 0</math> at <math>x = 0.129458</math> and <math>x = 0.222734</math> The graph of <math>g</math> is concave down on <math>(0.1295, 0.2227)</math> because <math>g''(x) &lt; 0</math> on this interval.</p> <p>(c) <math>g'(0.3) = -0.472161</math>  <math display="block">g(0.3) = 2 + \int_1^{0.3} g'(x) dx = 1.546007</math> An equation for the line tangent to the graph of <math>g</math> is <math>y = 1.546 - 0.472(x - 0.3)</math>.</p> <p>(d) <math>g''(x) &gt; 0</math> for <math>0.3 &lt; x &lt; 1</math> Therefore the line tangent to the graph of <math>g</math> at <math>x = 0.3</math> lies below the graph of <math>g</math> for <math>0.3 &lt; x &lt; 1</math>.</p>	<p>2 : <math>\begin{cases} 1 : \text{sets } g'(x) = 0 \\ 1 : \text{answer} \end{cases}</math></p> <p>2 : <math>\begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}</math></p> <p>4 : <math>\begin{cases} 1 : g'(0.3) \\ 1 : \text{integral expression} \\ 1 : g(0.3) \\ 1 : \text{equation} \end{cases}</math></p> <p>1 : answer with reason</p>
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**Question 3**

$t$	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time  $t = 0$ . During the time interval  $0 \leq t \leq 12$  hours, water is pumped into the pool at the rate  $P(t)$  cubic feet per hour. The table above gives values of  $P(t)$  for selected values of  $t$ . During the same time interval, water is leaking from the pool at the rate  $R(t)$  cubic feet per hour, where  $R(t) = 25e^{-0.05t}$ . (Note: The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is given by  $V = \pi r^2 h$ .)

- (a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval  $0 \leq t \leq 12$  hours. Show the computations that lead to your answer.
- (b) Calculate the total amount of water that leaked out of the pool during the time interval  $0 \leq t \leq 12$  hours.
- (c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time  $t = 12$  hours. Round your answer to the nearest cubic foot.
- (d) Find the rate at which the volume of water in the pool is increasing at time  $t = 8$  hours. How fast is the water level in the pool rising at  $t = 8$  hours? Indicate units of measure in both answers.

(a)  $\int_0^{12} P(t) dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

2 :  $\begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$

(b)  $\int_0^{12} R(t) dt = 225.594 \text{ ft}^3$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c)  $1000 + \int_0^{12} P(t) dt - \int_0^{12} R(t) dt = 1434.406$

1 : answer

At time  $t = 12$  hours, the volume of water in the pool is approximately 1434  $\text{ft}^3$ .

(d)  $V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241 \text{ or } 43.242 \text{ ft}^3/\text{hr}$

$V = \pi(12)^2 h$

$\frac{dV}{dt} = 144\pi \frac{dh}{dt}$

$\frac{dh}{dt} \Big|_{t=8} = \frac{1}{144\pi} \cdot \frac{dV}{dt} \Big|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft/hr}$

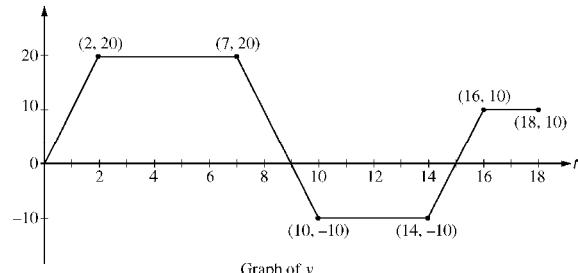
4 :  $\begin{cases} 1 : V'(8) \\ 1 : \text{equation relating } \frac{dV}{dt} \text{ and } \frac{dh}{dt} \\ 1 : \frac{dh}{dt} \Big|_{t=8} \\ 1 : \text{units of ft}^3/\text{hr and ft/hr} \end{cases}$

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**Question 4**

A squirrel starts at building  $A$  at time  $t = 0$  and travels along a straight wire connected to building  $B$ . For  $0 \leq t \leq 18$ , the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

- (a) At what times in the interval  $0 < t < 18$ , if any, does the squirrel change direction? Give a reason for your answer.
- (b) At what time in the interval  $0 \leq t \leq 18$  is the squirrel farthest from building  $A$ ? How far from building  $A$  is the squirrel at this time?
- (c) Find the total distance the squirrel travels during the time interval  $0 \leq t \leq 18$ .
- (d) Write expressions for the squirrel's acceleration  $a(t)$ , velocity  $v(t)$ , and distance  $x(t)$  from building  $A$  that are valid for the time interval  $7 < t < 10$ .



- (a) The squirrel changes direction whenever its velocity changes sign. This occurs at  $t = 9$  and  $t = 15$ .

2 :  $\begin{cases} 1 : t\text{-values} \\ 1 : \text{explanation} \end{cases}$

- (b) Velocity is 0 at  $t = 0$ ,  $t = 9$ , and  $t = 15$ .

2 :  $\begin{cases} 1 : \text{identifies candidates} \\ 1 : \text{answers} \end{cases}$

$t$	position at time $t$
0	0
9	$\frac{9+5}{2} \cdot 20 = 140$
15	$140 - \frac{6+4}{2} \cdot 10 = 90$
18	$90 + \frac{3+2}{2} \cdot 10 = 115$

The squirrel is farthest from building  $A$  at time  $t = 9$ ; its greatest distance from the building is 140.

- (c) The total distance traveled is  $\int_0^{18} |v(t)| dt = 140 + 50 + 25 = 215$ .

1 : answer

(d) For  $7 < t < 10$ ,  $a(t) = \frac{20 - (-10)}{7 - 10} = -10$

4 :  $\begin{cases} 1 : a(t) \\ 1 : v(t) \\ 2 : x(t) \end{cases}$

$$v(t) = 20 - 10(t - 7) = -10t + 90$$

$$x(7) = \frac{7+5}{2} \cdot 20 = 120$$

$$x(t) = x(7) + \int_7^t (-10u + 90) du$$

$$= 120 + \left( -5u^2 + 90u \right) \Big|_{u=7}^{u=t}$$

$$= -5t^2 + 90t - 265$$

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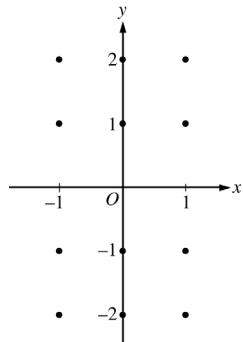
**Question 5**

Consider the differential equation  $\frac{dy}{dx} = \frac{x+1}{y}$ .

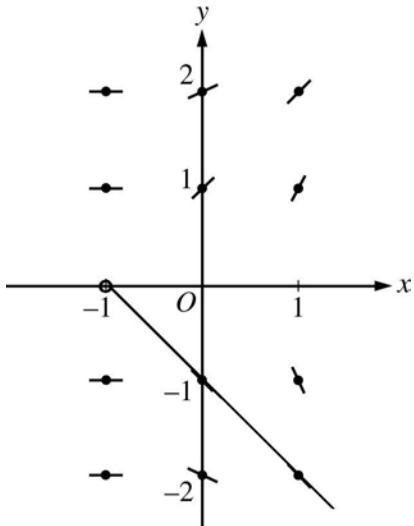
- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for  $-1 < x < 1$ , sketch the solution curve that passes through the point  $(0, -1)$ .

**(Note: Use the axes provided in the exam booklet.)**

- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the  $xy$ -plane for which  $y \neq 0$ . Describe all points in the  $xy$ -plane,  $y \neq 0$ , for which  $\frac{dy}{dx} = -1$ .
- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = -2$ .



(a)



$$3 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \\ 1 : \text{solution curve through } (0, -1) \end{cases}$$

(b)  $-1 = \frac{x+1}{y} \Rightarrow y = -x - 1$

$\frac{dy}{dx} = -1$  for all  $(x, y)$  with  $y = -x - 1$  and  $y \neq 0$

(c)  $\int y \, dy = \int (x+1) \, dx$

$$\frac{y^2}{2} = \frac{x^2}{2} + x + C$$

$$\frac{(-2)^2}{2} = \frac{0^2}{2} + 0 + C \Rightarrow C = 2$$

$$y^2 = x^2 + 2x + 4$$

Since the solution goes through  $(0, -2)$ ,  $y$  must be negative. Therefore  $y = -\sqrt{x^2 + 2x + 4}$ .

1 : description

$$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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**Question 6**

Two particles move along the  $x$ -axis. For  $0 \leq t \leq 6$ , the position of particle  $P$  at time  $t$  is given by  $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$ , while the position of particle  $R$  at time  $t$  is given by  $r(t) = t^3 - 6t^2 + 9t + 3$ .

- (a) For  $0 \leq t \leq 6$ , find all times  $t$  during which particle  $R$  is moving to the right.
- (b) For  $0 \leq t \leq 6$ , find all times  $t$  during which the two particles travel in opposite directions.
- (c) Find the acceleration of particle  $P$  at time  $t = 3$ . Is particle  $P$  speeding up, slowing down, or doing neither at time  $t = 3$ ? Explain your reasoning.
- (d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval  $1 \leq t \leq 3$ .

(a)  $r'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$   
 $r'(t) = 0$  when  $t = 1$  and  $t = 3$   
 $r'(t) > 0$  for  $0 < t < 1$  and  $3 < t < 6$   
 $r'(t) < 0$  for  $1 < t < 3$

2 :  $\begin{cases} 1 : r'(t) \\ 1 : \text{answer} \end{cases}$

Therefore  $R$  is moving to the right for  $0 < t < 1$  and  $3 < t < 6$ .

(b)  $p'(t) = -2 \cdot \frac{\pi}{4} \sin\left(\frac{\pi}{4}t\right)$   
 $p'(t) = 0$  when  $t = 0$  and  $t = 4$   
 $p'(t) < 0$  for  $0 < t < 4$   
 $p'(t) > 0$  for  $4 < t < 6$

3 :  $\begin{cases} 1 : p'(t) \\ 1 : \text{sign analysis for } p'(t) \\ 1 : \text{answer} \end{cases}$

Therefore the particles travel in opposite directions for  $0 < t < 1$  and  $3 < t < 4$ .

(c)  $p''(t) = -2 \cdot \frac{\pi}{4} \cdot \frac{\pi}{4} \cos\left(\frac{\pi}{4}t\right)$   
 $p''(3) = -2\left(\frac{\pi}{4}\right)^2 \cos\left(\frac{3\pi}{4}\right) = \frac{\pi^2}{8} \cdot \frac{\sqrt{2}}{2} > 0$   
 $p'(3) < 0$

2 :  $\begin{cases} 1 : p''(3) \\ 1 : \text{answer with reason} \end{cases}$

Therefore particle  $P$  is slowing down at time  $t = 3$ .

(d)  $\frac{1}{2} \int_1^3 |p(t) - r(t)| dt$

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$