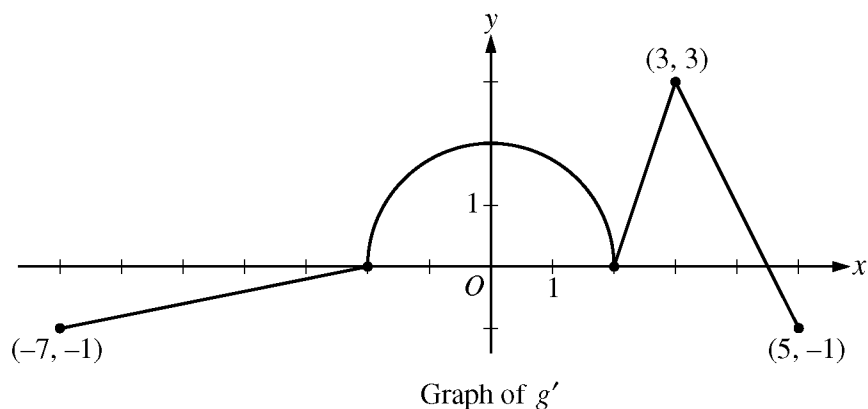


**AP<sup>®</sup> CALCULUS AB  
2010 SCORING GUIDELINES**

**Question 5**



The function  $g$  is defined and differentiable on the closed interval  $[-7, 5]$  and satisfies  $g(0) = 5$ . The graph of  $y = g'(x)$ , the derivative of  $g$ , consists of a semicircle and three line segments, as shown in the figure above.

- (a) Find  $g(3)$  and  $g(-2)$ .
- (b) Find the  $x$ -coordinate of each point of inflection of the graph of  $y = g(x)$  on the interval  $-7 < x < 5$ . Explain your reasoning.
- (c) The function  $h$  is defined by  $h(x) = g(x) - \frac{1}{2}x^2$ . Find the  $x$ -coordinate of each critical point of  $h$ , where  $-7 < x < 5$ , and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(a)  $g(3) = 5 + \int_0^3 g'(x) dx = 5 + \frac{\pi \cdot 2^2}{4} + \frac{3}{2} = \frac{13}{2} + \pi$   
 $g(-2) = 5 + \int_0^{-2} g'(x) dx = 5 - \pi$

3 :  $\begin{cases} 1 : \text{uses } g(0) = 5 \\ 1 : g(3) \\ 1 : g(-2) \end{cases}$

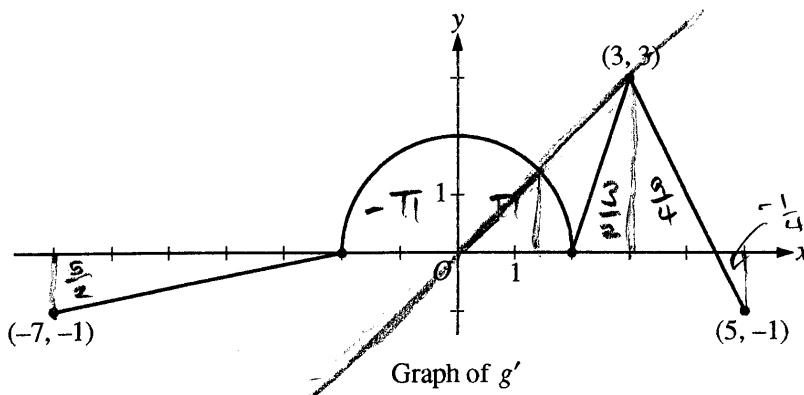
(b) The graph of  $y = g(x)$  has points of inflection at  $x = 0$ ,  $x = 2$ , and  $x = 3$  because  $g'$  changes from increasing to decreasing at  $x = 0$  and  $x = 3$ , and  $g'$  changes from decreasing to increasing at  $x = 2$ .

2 :  $\begin{cases} 1 : \text{identifies } x = 0, 2, 3 \\ 1 : \text{explanation} \end{cases}$

(c)  $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$   
 On the interval  $-2 \leq x \leq 2$ ,  $g'(x) = \sqrt{4 - x^2}$ .  
 On this interval,  $g'(x) = x$  when  $x = \sqrt{2}$ .  
 The only other solution to  $g'(x) = x$  is  $x = 3$ .  
 $h'(x) = g'(x) - x > 0$  for  $0 \leq x < \sqrt{2}$   
 $h'(x) = g'(x) - x \leq 0$  for  $\sqrt{2} < x \leq 5$   
 Therefore  $h$  has a relative maximum at  $x = \sqrt{2}$ , and  $h$  has neither a minimum nor a maximum at  $x = 3$ .

4 :  $\begin{cases} 1 : h'(x) \\ 1 : \text{identifies } x = \sqrt{2}, 3 \\ 1 : \text{answer for } \sqrt{2} \text{ with analysis} \\ 1 : \text{answer for 3 with analysis} \end{cases}$

NO CALCULATOR ALLOWED



Work for problem 5(a)

$$g(0) = 5$$

$$g(3) - g(0) = \int_0^3 g'(x) dx$$

$$g(3) = 5 + \int_0^3 g'(x) dx$$

$$g(3) = 5 + \left( \pi + \frac{\pi}{2} \right)$$

$$g(3) = \frac{13}{2} + \pi$$

$$g(-2) - g(0) = \int_0^{-2} g'(x) dx$$

$$g(-2) = 5 + \int_0^{-2} g'(x) dx$$

$$g(-2) = 5 + (-\pi)$$

$$g(-2) = 5 - \pi$$

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Continue problem 5 on page 13.

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Work for problem 5(b)

$$x = 0, 2, 3$$

$g'(x)$  switches from increasing to decreasing or from decreasing to increasing at these three  $x$ -values.

Work for problem 5(c)

$$h(x) = g(x) - \frac{1}{2}x^2$$

$$h'(x) = g'(x) - x$$

Where  $h'(x) = 0 \dots$

$$g'(x) - x = 0$$

$$g'(x) = x$$

$$x = \sqrt{2}, 3$$

$x = \sqrt{2}$  is a relative maximum for  $h(x)$  because  $h'(x)$  switches from positive to negative at this point.

$x = 3$  is not a relative extremum for  $h(x)$  because  $h'(x)$  does not switch signs.

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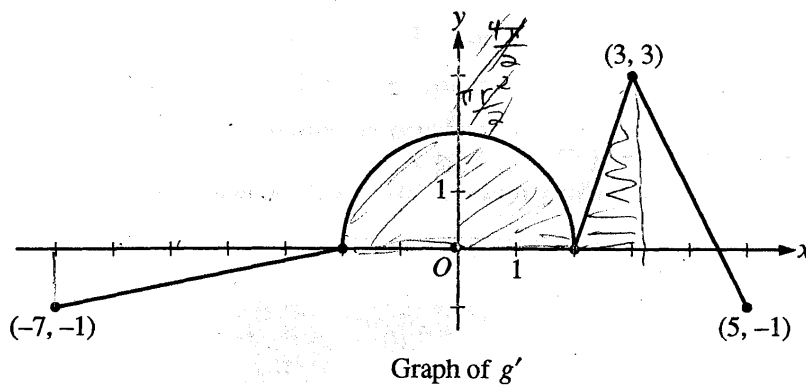
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5B

NO CALCULATOR ALLOWED



Work for problem 5(a)

$$g(3) = 5 + \int_0^3 g'(x) dx$$

$$= 5 + \left[ \left( \frac{1}{2}(3)(1) \right) + \left( \frac{\pi(1)^2}{4} \right) \right]$$

$$= 5 + \frac{3}{2} + \pi$$

$$g(3) = \frac{13}{2} + \pi$$

$$\begin{aligned} A &= \frac{\pi r^2}{4} \\ &= \frac{2^2 \pi}{4} \\ &= \pi \end{aligned}$$

$$g(-2) = 5 + \int_0^{-2} g'(x) dx$$

$$= 5 - \int_{-2}^0 g'(x) dx$$

$$g(-2) = 5 - \pi$$

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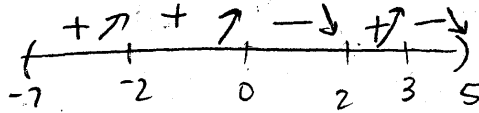
5B

NO CALCULATOR ALLOWED

Work for problem 5(b)

$$g''(x) = 0 \text{ @ } x = 0$$

$$g''(x) = \text{DNE @ } x = 2, x = 3, x = -2$$



$g$  has a pt of inflection @  $x = 0, x = 2, x = 3$  b/c  $g'$  goes from inc to dec, or vice versa, at those pts.

F-4

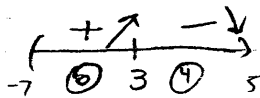
Work for problem 5(c)

$$g'(x) = x$$

$$h'(x) = g'(x) - x$$

$$0 = g'(x) - x$$

$$h'(x) = 0 \text{ @ } x = 3$$



$h$  has a critical pt @  $x = 3$ . At  $x = 3$   $h$  has a rel. max b/c  $h'$  goes from pos to neg at that pt.

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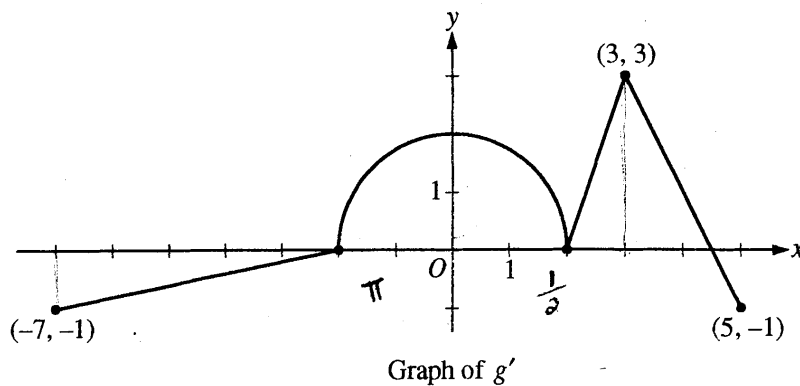
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5C

NO CALCULATOR ALLOWED



Work for problem 5(a)

$$g(3) = 5 + \int_0^3 g'(x) dx$$

$$5 + \left(\frac{1}{2} \cdot 3 \cdot 1\right) + \frac{1}{4} \pi 2^2$$

$$g(3) = 5 + \frac{3}{2} + \pi$$

$$g(-2) = 5 - \int_{-2}^0 g'(x) dx$$

$$\pi + \left(7 \cdot 1 \cdot \frac{1}{2}\right)$$

$$g(-2) = 5 - \frac{7}{2} + \pi$$

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Continue problem 5 on page 13.

Work for problem 5(b)

point of inf. when  $g''$  has a zero or when  $g'$  changes sign

$g'$  changes sign at  $x=0$ ,  $x=2$  and  $x=3$

$\therefore g$  has a point of inf. at  $x=0$ ,  $x=2$  and  $x=3$

Work for problem 5(c)

critical pt when  $h'(t) = 0$

$$h'(x) = g'(x) - x$$

critical point at  $x=3$

$h$	↑	↓
$h'$	+	-
	3	

$\therefore$  there is a rel. max when  $x=3$

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**AP<sup>®</sup> CALCULUS AB**  
**2010 SCORING COMMENTARY**

**Question 5**

**Overview**

This problem described a function  $g$  that is defined and differentiable on  $[-7, 5]$  and with  $g(0) = 5$ . The graph of  $y = g'(x)$  on  $[-7, 5]$  was given, consisting of three line segments and a semicircle. Part (a) asked for values of  $g(3)$  and  $g(-2)$ . These values are given by  $5 + \int_0^3 g'(x) dx$  and  $5 + \int_0^{-2} g'(x) dx$ , respectively, with the definite integrals computed using geometry and properties of definite integrals. Part (b) asked for the  $x$ -coordinates of points of inflection for the graph of  $y = g(x)$  on the interval  $-7 < x < 5$ . Students needed to reason graphically that these occur where the graph of  $g'$  changes from increasing to decreasing or vice versa.

Part (c) introduced a new function  $h(x) = g(x) - \frac{1}{2}x^2$  and asked for  $x$ -coordinates of critical points of  $h$  and for the classification of each critical point as the location of a relative minimum, relative maximum or neither. Students needed to find that  $h'(x) = g'(x) - x$  in order to determine the  $x$ -coordinates of critical points and apply a sign analysis of  $h'$  to classify these critical points.

**Sample: 5A**

**Score: 9**

The student earned all 9 points.

**Sample: 5B**

**Score: 6**

The student earned 6 points: 3 points in part (a), 2 points in part (b), and 1 point in part (c). In parts (a) and (b), the student's work is correct. In part (c) the student earned the first point for correctly computing  $h'(x)$ . Since  $x = \sqrt{2}$  is never identified, the student did not earn the second and third points. The fourth point was not earned since the student attempts to justify that  $x = 3$  is the  $x$ -coordinate of a relative maximum.

**Sample: 5C**

**Score: 4**

The student earned 4 points: 2 points in part (a), 1 point in part (b), and 1 point in part (c). In part (a) the student earned the first 2 points for correctly using  $g(0) = 5$  and computing  $g(3)$ . The third point was not earned since the value for  $g(-2)$  is incorrect. In part (b) the student earned the first point for correctly identifying all three  $x$ -coordinates of the points of inflection. The student's statement that " $g'$  changes sign" is not a justification for a point of inflection, so the second point was not earned. In part (c) the student earned the first point for correctly computing  $h'(x)$ . Since  $x = \sqrt{2}$  is never identified, the student did not earn the second and third points. The fourth point was not earned since the student attempts to justify that  $x = 3$  is the  $x$ -coordinate of a relative maximum.