Question 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

(a) $a(5) \approx \dfrac{v(10) - v(0)}{10 - 0} = \dfrac{0.3}{10} = 0.03 \text{ meters/sec}^2$	1 : answer
(b) $\int_0^{60} \lvert v(t)\rvert\, dt$ is the total distance, in meters, that Ben rides over the 60-second interval $t = 0$ to $t = 60$. $\int_0^{60} \lvert v(t)\rvert\, dt \approx 2.0 \cdot 10 + 2.3(40 - 10) + 2.5(60 - 40) = 139$ meters	$2 : \begin{cases} 1 : \text{meaning of integral} \\ 1 : \text{approximation} \end{cases}$
(c) Because $\dfrac{B(60) - B(40)}{60 - 40} = \dfrac{49 - 9}{20} = 2$, the Mean Value Theorem implies there is a time t, $40 < t < 60$, such that $v(t) = 2$.	$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{conclusion with justification} \end{cases}$
(d) $2L(t)L'(t) = 2B(t)B'(t)$ $L'(40) = \dfrac{B(40)v(40)}{L(40)} = \dfrac{9 \cdot 2.5}{\sqrt{144 + 81}} = \dfrac{3}{2}$ meters/sec	$3 : \begin{cases} 1 : \text{derivatives} \\ 1 : \text{uses } B'(t) = v(t) \\ 1 : \text{answer} \end{cases}$
	1 : units in (a) or (b)

NO CALCULATOR ALLOWED

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Work for problem 5(a)

$a(5) = \frac{v(10) - v(0)}{10 - 0} = \frac{2,3 - 2,0}{10} = 0,03$ meters/(second)^2

Work for problem 5(b)

$\int_0^{60} |v(t)|dt$ means a total distance, travelled by Ben during the time $= 60$ seconds, $0 < t < 60$.

$\int_0^{60} |v(t)|dt = v(0) \cdot \Delta t + v(10)\,\Delta t + v(40)\Delta t =$

$= 2 \cdot 10 + 2,3 \cdot 30 + 2,5 \cdot 20 = 139$ meters

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Work for problem 5(c)

Accoarding to Mean Value Teozem

$$\frac{B(60) - B(40)}{60-40} = \frac{49-9}{20} = 2 \, ;$$

$\Rightarrow$ There must be a time t when velecity is equal to 2 meters / second

Work for problem 5(d)

Is $(L(t))^2 = 12^2 + (B(t))^2$

$$L(t) = \sqrt{12^2 + (B(t))^2}$$

$$L'(t) = \frac{2B(t) \cdot B'(t)}{2\sqrt{12^2 + (B(t))^2}}$$

$$L'(40) = \frac{9 \cdot 2{,}5}{\sqrt{225}} = \frac{9 \cdot 2{,}5}{15} = \frac{3 \cdot 2{,}5}{5} = \frac{7{,}5}{5} =$$

$= 1{,}5$ meters / second

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Work for problem 5(a)

$a(5) = v'(5) = \frac{v(10)-v(0)}{10-0} = \frac{2.3-2}{10} = 0.03$

Work for problem 5(b)

the meaning of $\int_0^{60} |v(t)|\,dt$: The total distance Ben rides from $t=0$ to $t=60$, which is measured by meters.

Left Riemann Sum approximation of $\int_0^{60} |v(t)|\,dt$:

$\int_0^{60} |v(t)|\,dt = 2 \times 10 + 2.3 \times (40-10) + 2.5 \times (60-40)$
$= 20 + 69 + 50$
$= 139$

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NO CALCULATOR ALLOWED

Work for problem 5(c)

uncertain. the total change from $t=40$ to $t=60$ is increase. If the velocity is 2 meters per second, there will be a decrease in $40 \le t \le 60$.

We can't find the exact change between $40 \le t \le 60$, so there is uncertainly a velocity is 2 meters per second

Work for problem 5(d)

$L(t)^2 = 12^2 + B(t)^2$ when $t=40$

$2L(40)\frac{dL}{dt} = 144 + 2B(40)\frac{dB}{dt}$ and $L(40) = \sqrt{144+B(40)^2}$

$= \sqrt{144+81}$

$2\times 15\frac{dL}{dt} = 144 + 2\times 9\frac{dB}{dt}$

$= \sqrt{225}$

$\frac{dL}{dt} = \frac{144+45}{30}$

$= 15$

$= 63$ meters per second

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Work for problem 5(a)

$a(t)$ at $t=5$: $\dfrac{v(10)-v(0)}{10-0}$

$= \dfrac{2.3-2.0}{10}$

$= \dfrac{.3}{10}$

$\boxed{= .03 \text{ m}^2/\text{second}}$

Work for problem 5(b)

$\int_0^{60} |v(t)|\,dt$ would be the total distance covered by Ben, whether going forwards or backwards.

$136 \times 30 = 4080$

$\int_0^{60} |v(t)|\,dt \approx 10(100) + 30(136) + 20(9)$

$= 1000 + 4080 + 180$

$= \boxed{5260 \text{ meters}}$

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NO CALCULATOR ALLOWED

Work for problem 5(c)

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$= \frac{49-9}{60-40}$

$= \frac{40}{20}$

$= \boxed{2\ m/s}$

According to the mean value theorem, there is a time $t=c$ in which Ben's velocity is 2 meters per second.

Work for problem 5(d)

$(L(t))^2 = 12^2 + (B(t))^2$

$(L(t))^2 = 144 + (B(40))^2$

$(L(t))^2 = 144 + 9^2$

$\sqrt{(L(t))^2} = \sqrt{225}$

$L(t) = 15$

$\frac{d}{dx}$

$\begin{array}{r} 144 \\ +\ 81 \\ \hline 225 \end{array}$

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Question 5

Sample: 5A
Score: 9

The student earned all 9 points. In part (d) the student solves for $L(t)$ prior to differentiating.

Sample: 5B
Score: 6

The student earned 6 points: 1 point in part (a), 2 points in part (b), no points in part (c), 2 points in part (d), and the units point. In parts (a) and (b) the student's work is correct. In part (c) the student's work is incorrect. In part (d) the student differentiates the expression incorrectly. The student uses $B'(40) = v(40) = 2.5,$ so the second point was earned. Because the student's derivative is of the form $LL' = BB' + C,$ where C is a constant, the student was eligible for the answer point. The student's answer is consistent with the expression presented, so the answer point was earned. The student earned the units point, because "meters" are mentioned in the interpretation of the meaning of the integral in part (b).

Sample: 5C
Score: 4

The student earned 4 points: 1 point in part (a), no points in part (b), 2 points in part (c), no points in part (d), and the units point. In parts (a) and (c) the student's work is correct. In part (b) the student does not mention the time interval, and the approximation is incorrect. In part (d) the student's work is incorrect. The student presents the correct units in part (b), so the units point was earned. The units in part (a) are incorrect.