



AP[®] Calculus AB 2011 Scoring Guidelines Form B

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AP[®] CALCULUS AB
2011 SCORING GUIDELINES (Form B)

Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S , where $S(t)$ is measured in millimeters and t is measured in days for $0 \leq t \leq 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

- (a) According to the model, what is the height of the water in the can at the end of the 60-day period?
- (b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.
- (c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.
- (d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M , where $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \leq t \leq 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \leq t \leq 60$ to justify that there exists a time t , $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

(a) $S(60) = \int_0^{60} S'(t) dt = 171.813 \text{ mm}$

3 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\frac{S(60) - S(0)}{60} = 2.863 \text{ or } 2.864 \text{ mm/day}$

1 : answer

(c) $V(t) = 100\pi S(t)$
 $V'(7) = 100\pi S'(7) = 602.218$

2 : $\begin{cases} 1 : \text{relationship between } V \text{ and } S \\ 1 : \text{answer} \end{cases}$

The volume of water in the can is increasing at a rate of $602.218 \text{ mm}^3/\text{day}$.

(d) $D(0) = -0.675 < 0$ and $D(60) = 69.37730 > 0$

2 : $\begin{cases} 1 : \text{considers } D(0) \text{ and } D(60) \\ 1 : \text{justification} \end{cases}$

Because D is continuous, the Intermediate Value Theorem implies that there is a time t , $0 < t < 60$, at which $D(t) = 0$. At this time, the heights of water in the two cans are changing at the same rate.

1 : units in (b) or (c)

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Question 2

A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \frac{600t}{t+3} & \text{for } 0 \leq t \leq 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

- (a) Is r continuous at $t = 5$? Show the work that leads to your answer.
- (b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.
- (c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.
- (d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

(a) $\lim_{t \rightarrow 5^-} r(t) = \lim_{t \rightarrow 5^-} \left(\frac{600t}{t+3} \right) = 375 = r(5)$
 $\lim_{t \rightarrow 5^+} r(t) = \lim_{t \rightarrow 5^+} (1000e^{-0.2t}) = 367.879$

Because the left-hand and right-hand limits are not equal, r is not continuous at $t = 5$.

2 : conclusion with analysis

(b) $\frac{1}{8} \int_0^8 r(t) dt = \frac{1}{8} \left(\int_0^5 \frac{600t}{t+3} dt + \int_5^8 1000e^{-0.2t} dt \right)$
 $= 258.052$ or 258.053

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$

(c) $r'(3) = 50$
 The rate at which water is draining out of the tank at time $t = 3$ hours is increasing at 50 liters/hour².

2 : $\begin{cases} 1 : r'(3) \\ 1 : \text{meaning of } r'(3) \end{cases}$

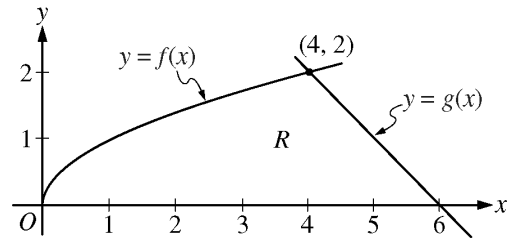
(d) $12,000 - \int_0^A r(t) dt = 9000$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

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Question 3

The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x -axis and the graphs of f and g , as shown in the figure above.



- (a) Find the area of R .
- (b) The region R is the base of a solid. For each y , where $0 \leq y \leq 2$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.
- (c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g . Find the coordinates of point P .

(a)
$$\text{Area} = \int_0^4 \sqrt{x} \, dx + \frac{1}{2} \cdot 2 \cdot 2 = \frac{2}{3}x^{3/2} \Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b)
$$\begin{aligned} y = \sqrt{x} &\Rightarrow x = y^2 \\ y = 6 - x &\Rightarrow x = 6 - y \end{aligned}$$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Width = $(6 - y) - y^2$

Volume = $\int_0^2 2y(6 - y - y^2) \, dy$

(c)
$$g'(x) = -1$$

Thus a line perpendicular to the graph of g has slope 1.

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$$

The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

3 : $\begin{cases} 1 : f'(x) \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

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Question 4

Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

- (a) Find the x -coordinate of the critical point of f . Determine whether the point is a relative maximum, a relative minimum, or neither for the function f . Justify your answer.
- (b) Find all intervals on which the graph of f is concave down. Justify your answer.
- (c) Given that $f(1) = 2$, determine the function f .

- (a) $f'(x) = 0$ at $x = 4$
 $f'(x) > 0$ for $0 < x < 4$
 $f'(x) < 0$ for $x > 4$
 Therefore f has a relative maximum at $x = 4$.

3 : $\begin{cases} 1 : x = 4 \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$

- (b) $f''(x) = -x^{-3} + (4 - x)(-3x^{-4})$
 $= -x^{-3} - 12x^{-4} + 3x^{-3}$
 $= 2x^{-4}(x - 6)$
 $= \frac{2(x - 6)}{x^4}$
 $f''(x) < 0$ for $0 < x < 6$

3 : $\begin{cases} 2 : f''(x) \\ 1 : \text{answer with justification} \end{cases}$

The graph of f is concave down on the interval $0 < x < 6$.

- (c) $f(x) = 2 + \int_1^x (4t^{-3} - t^{-2}) dt$
 $= 2 + \left[-2t^{-2} + t^{-1} \right]_{t=1}^{t=x}$
 $= 3 - 2x^{-2} + x^{-1}$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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Question 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t , measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t .

- (a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.
- (b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)| dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)| dt$ using a left Riemann sum with the subintervals indicated by the data in the table.
- (c) For $40 \leq t \leq 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.
- (d) A light is directly above the western end of the track. Ben rides so that at time t , the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

(a) $a(5) \approx \frac{v(10) - v(0)}{10 - 0} = \frac{0.3}{10} = 0.03 \text{ meters/sec}^2$

1 : answer

- (b) $\int_0^{60} |v(t)| dt$ is the total distance, in meters, that Ben rides over the 60-second interval $t = 0$ to $t = 60$.

2 : $\left\{ \begin{array}{l} 1 : \text{meaning of integral} \\ 1 : \text{approximation} \end{array} \right.$

$$\int_0^{60} |v(t)| dt \approx 2.0 \cdot 10 + 2.3(40 - 10) + 2.5(60 - 40) = 139 \text{ meters}$$

- (c) Because $\frac{B(60) - B(40)}{60 - 40} = \frac{49 - 9}{20} = 2$, the Mean Value Theorem implies there is a time t , $40 < t < 60$, such that $v(t) = 2$.

2 : $\left\{ \begin{array}{l} 1 : \text{difference quotient} \\ 1 : \text{conclusion with justification} \end{array} \right.$

(d) $2L(t)L'(t) = 2B(t)B'(t)$
 $L'(40) = \frac{B(40)v(40)}{L(40)} = \frac{9 \cdot 2.5}{\sqrt{144 + 81}} = \frac{3}{2} \text{ meters/sec}$

3 : $\left\{ \begin{array}{l} 1 : \text{derivatives} \\ 1 : \text{uses } B'(t) = v(t) \\ 1 : \text{answer} \end{array} \right.$

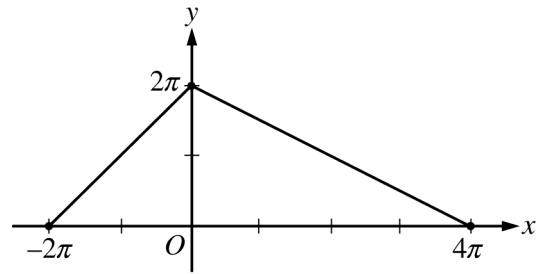
1 : units in (a) or (b)

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Question 6

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$

whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.



Graph of g

- (a) Find $\int_{-2\pi}^{4\pi} f(x) dx$. Show the computations that lead to your answer.
- (b) Find all x -values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.
- (c) Let $h(x) = \int_0^{3x} g(t) dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

$$\begin{aligned} \text{(a)} \quad \int_{-2\pi}^{4\pi} f(x) dx &= \int_{-2\pi}^{4\pi} \left(g(x) - \cos\left(\frac{x}{2}\right) \right) dx \\ &= 6\pi^2 - \left[2\sin\left(\frac{x}{2}\right) \right]_{x=-2\pi}^{x=4\pi} \\ &= 6\pi^2 \end{aligned}$$

2 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\text{(b)} \quad f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right) = \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi < x < 0 \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0 < x < 4\pi \end{cases}$$

4 : $\begin{cases} 1 : \frac{d}{dx}\left(\cos\left(\frac{x}{2}\right)\right) \\ 1 : g'(x) \\ 1 : x = 0 \\ 1 : x = \pi \end{cases}$

$f'(x)$ does not exist at $x = 0$.

For $-2\pi < x < 0$, $f'(x) \neq 0$.

For $0 < x < 4\pi$, $f'(x) = 0$ when $x = \pi$.

f has critical points at $x = 0$ and $x = \pi$.

$$\begin{aligned} \text{(c)} \quad h'(x) &= g(3x) \cdot 3 \\ h'\left(-\frac{\pi}{3}\right) &= 3g(-\pi) = 3\pi \end{aligned}$$

3 : $\begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$