AP® CALCULUS AB
2012 SCORING GUIDELINES

Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is $55°F$. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a) $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{67.9 - 61.8}{6}$
$= 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately $1.017\ °F$ per minute at time $t = 12$ minutes.

$2: \begin{cases} 1: \text{estimate} \\ 1: \text{interpretation with units} \end{cases}$

(b) $\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by $16\ °F$ over the interval from $t = 0$ to $t = 20$ minutes.

$2: \begin{cases} 1: \text{value} \\ 1: \text{interpretation with units} \end{cases}$

(c) $$\begin{aligned} \frac{1}{20}\int_0^{20} W(t)\,dt &\approx \frac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15)) \\ &= \frac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9) \\ &= \frac{1}{20} \cdot 1215.8 = 60.79 \end{aligned}$$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

$3: \begin{cases} 1: \text{left Riemann sum} \\ 1: \text{approximation} \\ 1: \text{underestimate with reason} \end{cases}$

(d) $$\begin{aligned} W(25) &= 71.0 + \int_{20}^{25} W'(t)\,dt \\ &= 71.0 + 2.043155 = 73.043 \end{aligned}$$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) \approx \frac{67.9-61.8}{15-9} = \boxed{1.0167\ °F/min}$$

At $t=12$, the temperature of the water in the tub is increasing at the rate of 1.067 °F/min.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} W'(t)dt = W(20) - W(0) = 71.0 - 55.0 = \boxed{16°F}$$

$\int_0^{20} W'(t)dt$ is the difference in temperature in °F of the water in the tub at $t = 20$ and $t=0$.

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(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$\int_0^{20} W(t)\,dt \approx 4(55.0) + 5(57.1) + 6(61.8) + 5(67.9)$$
$$= 1215.8$$
$$\frac{1}{20}\int_0^{20} W(t)\,dt = \boxed{60.79\,°F}$$

As the function W(t) is strictly increasing, the approximation rectangles of the left Riemann sum fall below the curve. Thus the approximation is an $\boxed{\text{underestimate.}}$

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$$\int_{20}^{25} W'(t) = W(25) - W(20) = 2.043$$
$$W(25) - 71.0 = 2.043$$
$$W(25) = \boxed{73.0432\,°F}$$

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1B.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) = \frac{W(15)-W(9)}{15-9} = \frac{67.9-61.8}{6} = \frac{6.1}{6} = 1.016\ \frac{°F}{min}$$

At time $t=12$, the water is being heated at a rate of $1.016\frac{°F}{min.}$

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} W'(t)\,dt = W(20) - W(0)$$
$$= 71.0 - 55.0 = 16.0 \text{ degrees Fahrenheit}$$

$\int_0^{20} W'(t)\,dt$ represents the change in water temperature in degrees Fahrenheit from time $t=0$ to $t=20$

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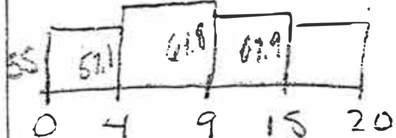
Continue problem 1 on page 5.

1B2

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.



$\frac{1}{20}\left[55(4-0)+57.1(9-4)+61.8(15-9)+67.9(20-15)\right]$

$\frac{1}{20}\left[55(4)+57.1(5)+61.8(6)+67.9(5)\right]$

$\frac{1}{20}\left[1215.8\right] = 60.79$

This approximation underestimates the average temp over the 20 minutes because $W''(t) > 0$. Concave up underestimates

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$W'(25) = 0.4\sqrt{25}\cos(.06(25))$

$= 0.141$

$y - 71.0 = 0.141(x-20)$

$y = 0.141x - 2.82 + 71$

$y = 0.141x + 68.18$

$W(25) = 0.141(25) + 68.18$

$= 71.705$

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(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$W'(12) \approx \frac{W(t)_2 - W(t)_1}{t_2 - t_1}$$

$$\approx \frac{W(9) - W(4)}{9-4}$$

$$\approx \frac{61.8 - 57.1}{9-4} \approx \frac{4.7}{5} \approx \boxed{.94 \text{ degrees/min}}$$

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\int_0^{20} W'(t)dt = W(t)\Big]_0^{20}$$

$$W(20) - W(0)$$

$$71 - 55 = \boxed{16 \text{ degrees Fahrenheit}}$$

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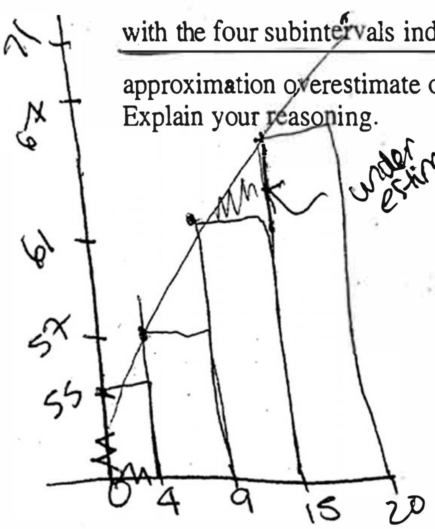
1C2

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\boxed{\frac{1}{20}\int_0^{20} W(t)\,dt}$. Use a $\boxed{\text{left}}$ Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.



~~+ (20-15)(71)~~

$(4-0)(55) + (9-4)(57.1) + (15-9)(61.2)$
$+ (20-15)(67.9)$

$220 + 285.5 + 370.8 + 339.5$

$\frac{1215.8}{20} \Rightarrow$ 60.79 avg temp

☆ ~~This~~ This approximation underestimates the average temperature as seen in the graph above. Since the height of the rectangles are being cut short, the estimate is smaller than the actual.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

20.43154699

20°F

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AP® CALCULUS AB
2012 SCORING COMMENTARY

Question 1

Overview

This problem involved a function W that models the temperature, in degrees Fahrenheit, of water in a tub. Values of $W(t)$ at selected times between $t = 0$ and $t = 20$ minutes are given in a table. Part (a) asked students for an approximation to the derivative of the function W at time $t = 12$ and for an interpretation of the answer. Students should have recognized this derivative as the rate at which the temperature of the water in the tub is increasing at time $t = 12,$ in degrees Fahrenheit per minute. Because $t = 12$ falls between the values presented in the table, students should have constructed a difference quotient using the temperature values across the smallest time interval containing $t = 12$ that is supported by the table. Part (b) asked students to evaluate the definite integral $\int_0^{20} W'(t)\,dt$ and to interpret the meaning of this definite integral. Students should have applied the Fundamental Theorem of Calculus and used values from the table to compute $W(20) - W(0).$ Students should have recognized this as the total change in the temperature of the water, in degrees Fahrenheit, over the 20-minute time interval. In part (c) students were given the expression for computing the average temperature of the water over the 20-minute time period and were asked to use a left Riemann sum with the four intervals given by the table to obtain a numerical approximation for this value. Students were asked whether this approximation overestimates or underestimates the actual average temperature. Students should have recognized that for a strictly increasing function, the left Riemann sum will underestimate the true value of a definite integral. In part (d) students were given the symbolic first derivative $W'(t)$ of the function W that models the temperature of the water over the interval $20 \le t \le 25,$ and were asked to use this expression to determine the temperature of the water at time $t = 25.$ This temperature is computed using the expression $W(25) = W(20) + \int_{20}^{25} W'(t)\,dt,$ where $W(20) = 71$ is given in the table.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and no points in part (d). In parts (a) and (b) the student's work is correct. In part (c) the student earned the left Riemann sum and approximation points. The student does not give a correct reason for "underestimates," so the last point in part (c) was not earned. In part (d) the student's work is incorrect.

Sample: 1C
Score: 3

The student earned 3 points: no points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In parts (a) and (d) the student's work is incorrect. In part (b) the student earned the value point. In part (c) the student earned the left Riemann sum and approximation points. The student does not give a correct reason for "underestimates," so the last point in part (c) was not earned.