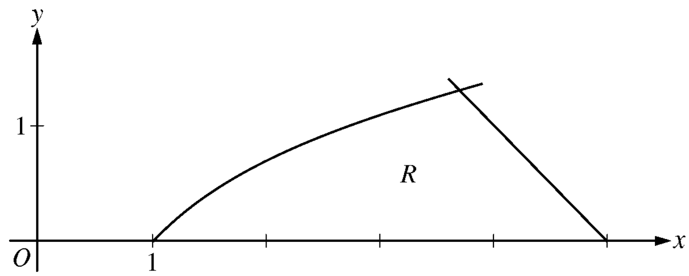
AP® CALCULUS AB
2012 SCORING GUIDELINES

Question 2



Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

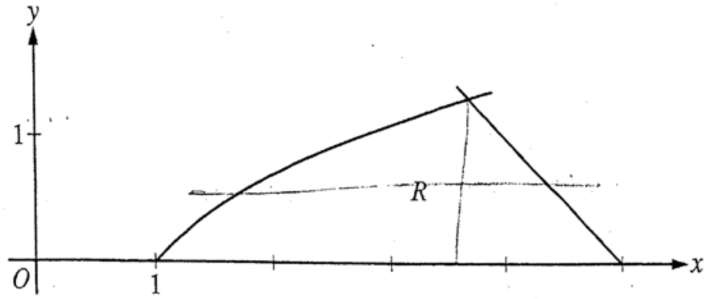
(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$\ln x = 5 - x \;\Rightarrow\; x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) Area $= \int_0^B \left(5 - y - e^y\right) dy$ $= 2.986$ (or 2.985) OR Area $= \int_1^A \ln x \, dx + \int_A^5 (5 - x)\, dx$ $= 2.986$ (or 2.985)	3 : { 1 : integrand 1 : limits 1 : answer
(b) Volume $= \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$	3 : { 2 : integrands 1 : expression for total volume
(c) $\int_0^k \left(5 - y - e^y\right) dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$	3 : { 1 : integrand 1 : limits 1 : equation

2A₁



2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

$e^y = x \qquad x = 5 - y$

(a) Find the area of R.

$$A = \int_0^{1.307} (5 - y - e^y)\,dy$$

$$= \boxed{2.986}$$

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Continue problem 2 on page 7.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

$$V = \int_{1}^{3.693} (\ln x)^2\, dx + \int_{3.693}^{5} (5-x)^2\, dx$$

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$$\int_{0}^{k} (5 - y - e^y)\, dy = \int_{k}^{1.307} (5 - y - e^y)\, dy$$

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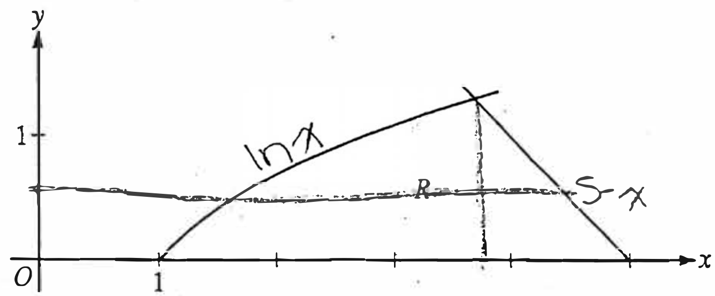

2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

   (a) Find the area of R.

~~$\text{Area} = \int_1^5 (\ln x) - (5-x)\,dx = 3.953 \text{ units}^2$~~

$$\text{Area} = \int_1^{3.693} (\ln x)\,dx + \int_{3.693}^{5} (5-x)\,dx$$

$$\text{Area} = 2.132 + 0.854 = 2.986 \text{ units}^2$$

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Continue problem 2 on page 7.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

$A = \text{side}^2$

$$V = \int_1^{3.693} \left[(\ln x)^2\right] dx + \int_{3.693}^{5} \left[(5-x)^2\right] dx$$

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$$\text{Area} = \int_1^{3.693} \left[\ln x - k\right] dx + \int_{3.693}^{5} \left[(5-x) - k\right] dx$$

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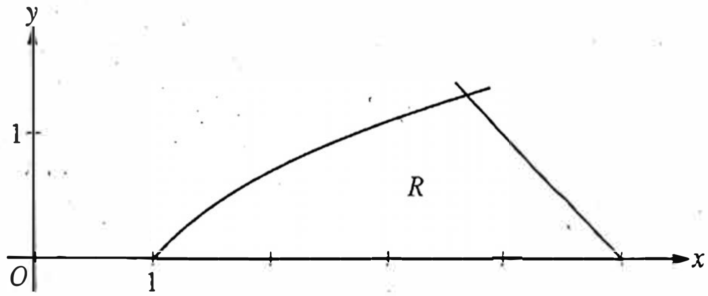

2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

$y = \ln x$ $\quad y = 5 - x$

$x = e^y$ $\quad x = 5 - y$

let $a = 1.30655$

$$\int_0^a [(5-y) - (e^y)]\,dy = 2.985$$

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Continue problem 2 on page 7.

 2C2

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

$$\int_0^a \left[(5-y)-(e^y)\right]^2 dy$$

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$$k = \frac{\int_0^a \left[(5-y)-(e^y)\right] dy}{a-0}$$

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AP® CALCULUS AB
2012 SCORING COMMENTARY

Question 2

Overview

Students were given the graph of a region R bounded below by the x-axis, on the left by the graph of $y = \ln x$, and on the right by the graph of the line $y = 5 - x$. In part (a) students were asked to find the area of R. This required an appropriate integral setup and evaluation. Students first needed to determine the intersection point, (A, B), of the two curves. The area could then be computed by solving each expression for x in terms of y and evaluating a single integral with respect to y. Alternatively, the area could be computed by evaluating a sum of two integrals with respect to x. In the first case, students would evaluate $\int_0^B \left(5 - y - e^y\right) dy$ and in the second case, $\int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx$. Part (b) asked for an expression involving one or more integrals that gives the volume of a solid whose base is the region R and whose cross sections perpendicular to the x-axis are squares. Students should have found the cross-sectional area function in terms of x, which is $(\ln x)^2$ on the interval $1 \le x \le A$ and $(5 - x)^2$ on the interval $A \le x \le 5$. These expressions are used as the integrands for two definite integrals with the corresponding endpoints, whose sum provides the desired expression. Part (c) asked for an equation involving one or more integrals whose solution gives the value k for which the line $y = k$ divides the region R into two smaller regions of equal area. Students should have first rewritten the equations for the curves as functions of x in terms of y. Two common solutions were setting the definite integral $\int_0^k \left(5 - y - e^y\right) dy$ equal to half the value of the area computed in part (a), and $\int_0^k \left(5 - y - e^y\right) dy = \int_k^B \left(5 - y - e^y\right) dy$.

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 3 points in part (a), 3 points in part (b), and no points in part (c). In part (a) the student's work is correct. The student works in terms of x in part (a) and correctly sums the two integrals for the area of R. In part (b) the student's work is correct. In part (c) the variable k (or an expression involving k) must be included in the limits of the integrals in order to be eligible for points.

Sample: 2C
Score: 3

The student earned 3 points: 3 points in part (a), no points in part (b), and no points in part (c). In part (a) the student's work is correct. In part (b) the student is not working with the correct cross section. In part (c) the variable k (or an expression involving k) must be included in the limits of the integrals in order to be eligible for points.