AP® CALCULUS AB
2013 SCORING GUIDELINES

Question 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

(a) $G'(5) = -24.588$ (or -24.587)

The rate at which gravel is arriving is decreasing by 24.588 (or 24.587) tons per hour per hour at time $t = 5$ hours.

$2 : \begin{cases} 1 : G'(5) \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\int_0^8 G(t)\,dt = 825.551$ tons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $G(5) = 98.140764 < 100$

At time $t = 5$, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed. Therefore, the amount of unprocessed gravel at the plant is decreasing at time $t = 5$.

$2 : \begin{cases} 1 : \text{compares } G(5) \text{ to } 100 \\ 1 : \text{conclusion} \end{cases}$

(d) The amount of unprocessed gravel at time t is given by $A(t) = 500 + \int_0^t (G(s) - 100)\,ds.$

$A'(t) = G(t) - 100 = 0 \Rightarrow t = 4.923480$

t	$A(t)$
0	500
4.92348	635.376123
8	525.551089

The maximum amount of unprocessed gravel at the plant during this workday is 635.376 tons.

$3 : \begin{cases} 1 : \text{considers } A'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$G(t) = 90 + 45\cos\frac{t^2}{18}$

$G'(t) = -5t\sin\left(\frac{t^2}{18}\right)$

$G'(5) = -24.588$ tons/hr^2

This means that the rate at which unprocessed gravel arrives at the processing plant is changing by -24.588 tons per hour per hour, or decreasing by 24.588 tons per hour per hour, at $t = 5$ hours.

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(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

$\int_0^8 \left[90 + 45\cos\left(\frac{t^2}{18}\right)\right]dt = 825.551$ tons

Continue problem 1 on page 5.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

Let $V(t)$ be the amount of unprocessed gravel.

$V'(t)$ is the rate at which the amount of unprocessed gravel is changing.

$V'(t) = G(t) - 100$

$V'(5) = G(5) - 100$

$= 90 + 45\cos\left(\frac{5^2}{18}\right) - 100$

$V'(5) = -1.859$

Since $V'(5)$ is negative, the amount of unprocessed gravel is decreasing at time $t = 5$ hours

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

$V'(t) = 0$ at $t = ?$

$0 = G(t) - 100$

$100 = 90 + 45\cos\left(\frac{t^2}{18}\right)$

$10 = 45\cos\left(\frac{t^2}{18}\right)$

$t = 4.923$

$V(t) - V(0) = \int_0^t (G(x) - 100)\,dx$

$V(t) = \int_0^t (G(x) - 100)\,dx + V(0)$

$V(0) = 500$

$V(4.923) = 635.376$

$V(8) = 525.551$

Since $V(t)$ is on a closed interval $[0, 8]$, the maximum amount must occur at an endpoint or at a critical value. After evaluating the amount of unprocessed gravel at $t = 0$, $t = 4.923$, and $t = 8$, the amount of unprocessed gravel is highest at $t = 4.923$, with 635.376 tons of unprocessed gravel.

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1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using ~~correct units,~~ interpret your answer in the context of the problem.

$G'(5) = -24.588$ tons/hour^2

$G'(5)$ represents the rate of change in tons/hour^2 of the rate at which unprocessed gravel arrives

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

$\int_0^8 90+45\cos\left(\frac{t^2}{18}\right)dt$

$= 825.551$ tons

Continue problem 1 on page 5.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

rate gravel is arriving $= G(5) = 98.141$ tons/hour

rate gravel is being processed $= 100$ tons/hour

The amount of unprocessed gravel at the plant at time t=5 hours is decreasing since the rate at which the gravel is being processed is greater than the rate at which it is arriving.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

$500 + \int_0^t 90 + 45\cos\left(\frac{t^2}{18}\right)$

$90 + 45\cos\left(\frac{t^2}{18}\right) = 100$

$t = 4.923$

$500 + \int_0^{4.923} 90 + 45\cos\left(\frac{t^2}{18}\right) dt$

1127.676 tons

this occurs ~~at the time~~ when the rate of the amount arriving equals the rate it is being processed

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1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$G'(t) = -5t\sin\left(\frac{t^2}{18}\right)$

$G'(5) = -5(5)\sin\left(\frac{25}{18}\right)$

$G'(5) = -25\sin\left(\frac{25}{18}\right) \approx$ -24.588 tons per hour^2

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

$\int_0^8 \left(90 + 45\cos\left(\frac{t^2}{18}\right)\right)dt =$ 825.551 tons

Continue problem 1 on page 5.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

$G(5) = 90 + 45\cos\left(\frac{5^2}{18}\right)$

$G(5) = 98.1408$

positive amount arriving

increasing

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

at $t=8$, max amount

~~$G(8) = 90 + 45\cos\left(\frac{8^2}{18}\right)$~~

at end of day

$\int 90 + 45\cos\left(\frac{t^2}{18}\right)dt = 825.551$ tons

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Question 1

Overview

This problem provided information related to the amount of gravel at a gravel processing plant during an eight-hour period. The function G, given by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, models the rate, in tons per hour, at which gravel arrives at the plant. The problem also stated that gravel is processed at a constant rate of 100 tons per hour. In part (a) students were asked to find $G'(5)$, the derivative of G at time $t = 5$. This value is negative, so students should have interpreted the absolute value of this number as the rate at which the rate of arrival of gravel at the plant is decreasing, in tons per hour per hour, at time $t = 5$. In part (b) students were asked to find the total amount of unprocessed gravel arriving at the plant over the eight-hour workday. Students should have evaluated the definite integral $\int_0^8 G(x)\,dx$, recognizing that integrating the rate at which gravel arrives over a time interval gives the net amount of gravel that arrived over that time interval. Part (c) asked whether the amount of unprocessed gravel at the plant is increasing or decreasing at time $t = 5$. Students determined whether the rate at which unprocessed gravel is arriving is greater than the rate at which gravel is being processed, i.e., whether $G(5) > 100$. Part (d) asked students to determine the maximum amount of unprocessed gravel at the plant during this workday. Because the amount of unprocessed gravel at the plant at time t is given by $A(t) = 500 + \int_0^t (G(s) - 100)\,ds$, students needed to identify the critical points of this function (where $G(t) = 100$) and to determine the global maximum on the interval $[0, 8]$. This could have been done by observing that there is a unique critical point on the interval, which is a maximum, and determining the amount of unprocessed gravel at the plant at that time, or by computing the amount of unprocessed gravel at this critical point and at the endpoints for comparison.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student presents a correct value for $G'(5)$ and earned the first point. The student does not address time $t = 5$ in the interpretation of the value, so the second point was not earned. In parts (b) and (c), the student's work is correct. In part (d) the first point was earned for considering where $G(t) = 100$. The student does not correctly determine the maximum amount of gravel, so the second point was not earned. A justification for a global maximum was not provided, so the third point was not earned.

Sample: 1C
Score: 3

The student earned 3 points: 1 point in part (a) and 2 points in part (b). In part (a) the student presents a correct value for $G'(5)$ and earned the first point. The student does not provide an interpretation of this value, so the second point was not earned. In part (b) the student's work is correct. In part (c) the student ignores the rate at

Question 1 (continued)

which gravel was being processed and did not earn either point. In part (d) the student again does not consider the rate at which gravel was being processed. The student did not earn any points in this part.