AP® CALCULUS AB
2013 SCORING GUIDELINES

Question 2

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) Solve $|v(t)| = 2$ on $2 \le t \le 4$.
$t = 3.128$ (or 3.127) and $t = 3.473$

$2 : \begin{cases} 1 : \text{considers } |v(t)| = 2 \\ 1 : \text{answer} \end{cases}$

(b) $s(t) = 10 + \int_0^t v(x)\,dx$

$s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$

$2 : \begin{cases} 1 : s(t) \\ 1 : s(5) \end{cases}$

(c) $v(t) = 0$ when $t = 0.536033,\ 3.317756$
$v(t)$ changes sign from negative to positive at time $t = 0.536033$.
$v(t)$ changes sign from positive to negative at time $t = 3.317756$.

Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).

$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$

(d) $v(4) = -11.475758 < 0,\ a(4) = v'(4) = -22.295714 < 0$

The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.

2 : conclusion with reason

2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

$|v(t)| = 2 \qquad 2 \le t \le 4$

$t = 3.128, 3.473$

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

$s(t) = 10 + \int_0^t v(x)\,dx$

$s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$

Continue problem 2 on page 7.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

$v(t) = 0$

$t = 0.536, 3.318$

the particle changes direction at 0.536 because $v(t) < 0$ for $(0, 0.536)$ and $v(t) > 0$ for $(0.536, 3.318)$. The particle changes direction at 3.318 because $v(t) > 0$ for $(0.536, 3.318)$ and $v(t) < 0$ for $t > 3.318$

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

$v(4) < 0$

$a(4) = v'(4) < 0$

The speed is increasing at $t = 4$ because both $v(4)$ and $a(4) = v'(4)$ are negative.

2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

$2 = -2 + (t^2+3t)^{6/5} - t^3$
$0 = -4 + (t^2+3t)^{6/5} - t^3$
$t = 3.128$

$t = 3.128$

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

$s(t) = 10 + \int_0^t v(x)\,dx$
$s(5) = 10 + \int_0^5 v(x)\,dx$
$s(5) = -9.207$

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Continue problem 2 on page 7.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

The particle changes direction when velocity changes sign. This occurs at time $t = 3.318$ as the velocity changes from positive to negative.

$0 = -2 + (t^2+3t)^{1.2} - t^3$

$t = 3.318$

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

$v(t) = -2 + (t^2+3t)^{1.2} - t^3$

$a(t) = 1.2\sqrt[5]{t^2+3t}\,(2t+3) - 3t^2$

$v(4) = -2 + (16+12)^{1.2} - 64$

$v(4) = -11.476$

$a(4) = 1.2\sqrt[5]{16+12}\,(8+3) - 48$

$a(4) = -22.296$

The speed of a particle is increasing if acceleration and velocity have the same sign and decreasing if acceleration and velocity have different signs. The speed of the particle is increasing at time $t = 4$ because, at that point, both the velocity and the acceleration are negative.

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2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

x y

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

$2 = -2 + (t^2+3t)^{6/5}$

$0 = (t^2+3t)^{6/5} - t^3 - 4$

→graph

at $t = 3.1276299$

$t = 3.128$

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

$s(t) = \int v(t) \rightarrow -t^2 + \frac{5}{11}\left(\frac{t^3}{3} + \frac{3t^2}{2}\right)^{11/5} - \frac{t^4}{4} + C$

$10 = 0 + \frac{5}{11}(0+0)^{11/5} - 0 + C \rightarrow C = 10$

$s(5) = -25 + 6829.210659 - 156.25 + 10$

$s(5) = 6657.961$

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Continue problem 2 on page 7.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

The particle changes direction when the velocity changes sign

$0 = -2 + (t^2+3t)^{6/5} - t^3$ → graph

$v(0) =$

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

$v(4) = -2 + (4^2+3(4))^{6/5} - 4^3$

$v(4) = -11.476$

$a(t) = \frac{6}{5}(t^2+3t)^{1/5} \cdot (2t+3) - 3t^2$

$a(4) = -22.296$

The speed is increasing because acceleration and velocity are both negative.

~~$s(4) = 1975.737$~~

AP® CALCULUS AB
2013 SCORING COMMENTARY

Question 2

Overview

This problem presented students with a particle in rectilinear motion during the time interval $0 \le t \le 5$. The particle's position at time $t = 0$ is given, and the velocity function v is provided. Part (a) asked students to determine the times when the speed of the particle is 2, which required determining where the velocity function is ± 2 or where the absolute value of the velocity function is 2. In part (b) students were asked to provide an integral expression for the position $s(t)$ and then to use this expression to find the position of the particle at time $t = 5$. Students should have recognized that the position is given by $s(t) = s(0) + \int_0^t v(x)\,dx$ and then evaluated $s(5)$ to determine the position at time $t = 5$. Part (c) asked students to determine all times t, $0 \le t \le 5$, at which the particle changes direction. Students needed to determine where $v(t)$ changes sign. In part (d) students were asked whether the speed of the particle is increasing or decreasing at time $t = 4$. Students should have evaluated both the velocity and the acceleration functions at time $t = 4$. Because $v(4) < 0$ and $a(4) < 0$, the speed of the particle is increasing.

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: no points in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student's work is incorrect. In part (b) the student's work is correct. In part (c) the student earned the point for considering $v(t) = 0$ and 1 point for a single correct answer with correct justification. In part (d) the student's work is correct.

Sample: 2C
Score: 3

The student earned 3 points: 1 point in part (c) and 2 points in part (d). In parts (a) and (b), the student's work is incorrect and did not earn any points. In part (c) the student earned the point for considering $v(t) = 0$. In part (d) the student's work is correct.