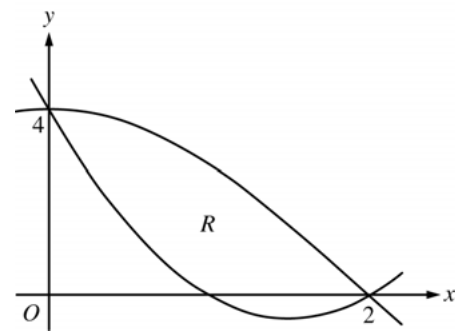
AP® CALCULUS AB
2013 SCORING GUIDELINES

Question 5



Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) $$\text{Area} = \int_0^2 [g(x) - f(x)]\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right] dx$$
$$= \left[4 \cdot \frac{4}{\pi}\sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right)\right]_0^2$$
$$= \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3}$$

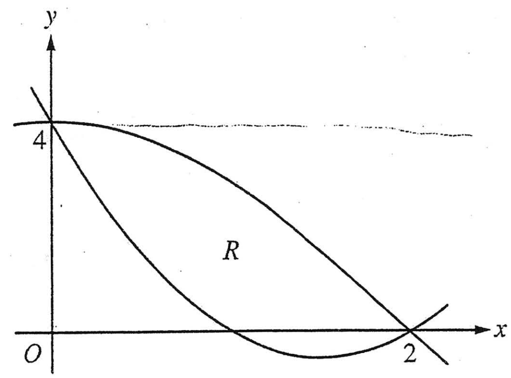
$$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

(b) $$\text{Volume} = \pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$$
$$= \pi\int_0^2 \left[\left(4 - \left(2x^2 - 6x + 4\right)\right)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$$

(c) $$\text{Volume} = \int_0^2 [g(x) - f(x)]^2\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right]^2 dx$$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$$



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

$$R = \int_0^2 \left(4\cos\left(\tfrac{1}{4}\pi x\right) - (2x^2 - 6x + 4)\right) dx$$

$$R = \left(\frac{16}{\pi}\sin\left(\tfrac{1}{4}\pi x\right) - \frac{2}{3}x^3 + 3x^2 - 4x\right)\Big|_0^2$$

$$R = \left(\frac{16}{\pi}\sin\left(\tfrac{1}{2}\pi\right) - \frac{16}{3} + 12 - 8\right) - \left(\frac{16}{\pi}\sin(0) - 0 + 0 - 0\right)$$

$$R = \frac{16}{\pi}(1) - \frac{16}{3} + 4$$

$$\boxed{R = \frac{16}{\pi} - \frac{4}{3}}$$

Do not write beyond this border.

Continue problem 5 on page 19.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

$$V = \pi \int_{0}^{2} \left(\left(4 - (2x^2 - 6x + 4)\right)^2 - \left(4 - 4\cos\left(\tfrac{1}{4}\pi x\right)\right)^2 \right) dx$$

$$\boxed{V = \pi \int_{0}^{2} \left((-2x^2 + 6x)^2 - \left(4 - 4\cos\left(\tfrac{1}{4}\pi x\right)\right)^2 \right) dx}$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$$V = \int_{0}^{2} \left(4\cos\left(\tfrac{1}{4}\pi x\right) - (2x^2 - 6x + 4) \right)^2 dx$$

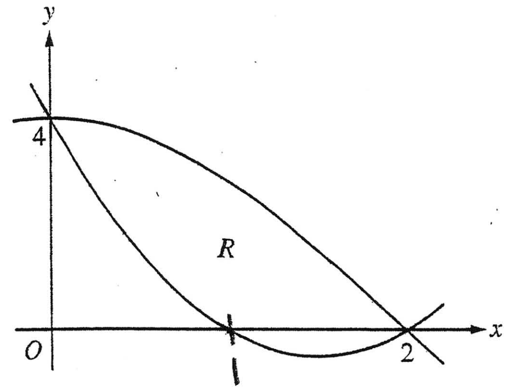
$$\boxed{V = \int_{0}^{2} \left(4\cos\left(\tfrac{1}{4}\pi x\right) - 2x^2 + 6x - 4 \right)^2 dx}$$

Do not write beyond this border.

Do not write beyond this border.

5B1

Do not write beyond this border.



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

$$\int_0^2 \left(4\cos\left(\tfrac{1}{4}\pi x\right) - \left(2x^2 - 6x + 4\right)\right) dx =$$

$$\int_0^2 \left(4\cos\left(\tfrac{1}{4}\pi x\right) - 2x^2 + 6x + 4\right) dx =$$

$$\pi \sin\left(\tfrac{1}{4}\pi x\right) - \tfrac{2}{3}x^3 + 3x^2 + 4x \Big|_0^2 =$$

$$\left(\pi\left(\sin\tfrac{\pi}{2}\right) - \tfrac{2}{3}(2)^3 + 3(2)^2 + 4(2)\right) =$$

$$\pi - \tfrac{16}{3} + 12 + 8 =$$

$$\boxed{\pi + \tfrac{44}{3} = R}$$

Do not write beyond this border.

Continue problem 5 on page 19.

NO CALCULATOR ALLOWED

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

V =

$$V = \pi \int_0^4 \left(\left[4 - \left(2x^2 - 6x + 4\right)\right]^2 - \left[4 - \left(4\cos\left(\tfrac{1}{4}\pi x\right)\right)\right]^2 \right) dx$$

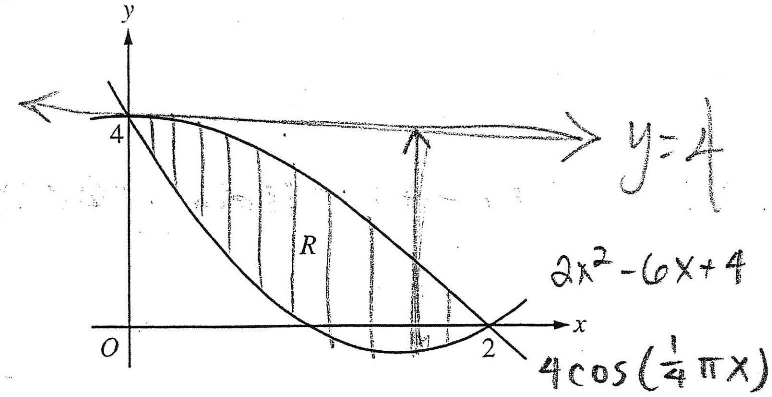
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$$\int_0^2 \left(\left(4\cos\left(\tfrac{1}{4}\pi x\right) - \left(2x^2 - 6x + 4\right)\right)^2 \right) dx$$

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

$\frac{1}{4}\pi(1) + x(0)$

$$\int_0^2 \left(\left(4\cos\left(\tfrac{1}{4}\pi x\right)\right) - \left(2x^2 - 6x + 4\right)\right) dx$$

(0,1)

$4\pi 4 \sin \frac{1}{4}\pi x$

$$16\pi \sin\left(\tfrac{1}{4}\pi x\right) - \tfrac{2}{3}x^3 - 3x^2 + 4x \Big]_0^2$$

$$16\pi \sin\left(\tfrac{1}{\cancel{4}_{2}}\pi \cancel{2}\right) - \tfrac{2}{3}\overset{8}{(2)^3} - 3(2)^2 + 8 - 0$$

$$16\pi \sin\left(\tfrac{\pi}{2}\right) - \tfrac{16}{3} - 12 + 8$$

$$\boxed{R = 16\pi - \tfrac{16}{3} - 4 \text{ units}^2}$$

Continue problem 5 on page 19.

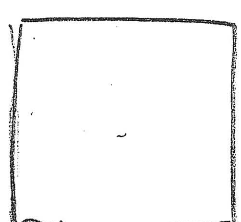
NO CALCULATOR ALLOWED

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

$$\int_0^2 \left(\left(4\cos\left(\tfrac{1}{4}\pi x\right) - 4\right)^2 - \left(\left(2x^2 - 6x + 4\right) - 4\right)^2 \right) dx$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$$\int_0^2 \left(\left(4\cos\left(\tfrac{1}{4}\pi x\right)\right)^2 - \left(2x^2 - 6x + 4\right)^2 \right) dx$$



$4\cos\left(\tfrac{1}{4}\pi x\right)$

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

AP® CALCULUS AB
2013 SCORING COMMENTARY

Question 5

Overview

Students were given the graph of a region R bounded below by the graph of the function f and above by the graph of the function g, where $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. In part (a) students were asked to find the area of R, requiring an appropriate integral setup and evaluation. Students needed to correctly evaluate $\int_0^2 (g(x) - f(x))\,dx$. Part (b) asked for an integral expression for the volume of the solid obtained by rotating the region R about the horizontal line $y = 4$. Students needed to set up an integral where the integrand represents a cross-sectional area of a circular disc with inner radius $(4 - g(x))$ and outer radius $(4 - f(x))$. This yielded the integral $\pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$. Part (c) asked for an integral expression for the volume of the solid whose base is the region R and whose cross sections perpendicular to the x-axis are squares. Here the required integrand was $(g(x) - f(x))^2$.

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), and 2 points in part (c). In part (a) the student earned the integrand point and polynomial antiderivative point. The answer point was not earned due to incorrect distribution of the negative sign. In part (b) the student earned both integrand points. The student uses incorrect limits and did not earn the third point. In part (c) the student's work is correct.

Sample: 5C
Score: 3

The student earned 3 points: 2 points in part (a) and 1 point in part (b). In part (a) the student earned the integrand point and polynomial antiderivative point. The answer point was not earned because of the incorrect distribution of the negative sign. In part (b) the student earned the first integrand point for one correct radius. Because the difference of the squares of the radii is reversed, the student did not earn the second integrand point. The constant of π is missing, so the third point was not earned. In part (c) the student did not earn the integrand point and therefore did not earn the limits and constant point.