AP® CALCULUS AB
2013 SCORING GUIDELINES

Question 6

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

(a) $\left.\frac{dy}{dx}\right|_{(x,\, y)=(1,\, 0)} = e^0\left(3\cdot 1^2 - 6\cdot 1\right) = -3$

An equation for the tangent line is $y = -3(x-1)$.

$f(1.2) \approx -3(1.2-1) = -0.6$

$$3 : \begin{cases} 1 : \frac{dy}{dx} \text{ at the point } (x, y) = (1, 0) \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$$

(b) $\frac{dy}{e^y} = \left(3x^2 - 6x\right)dx$

$\int \frac{dy}{e^y} = \int \left(3x^2 - 6x\right)dx$

$-e^{-y} = x^3 - 3x^2 + C$

$-e^{-0} = 1^3 - 3\cdot 1^2 + C \Rightarrow C = 1$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln\left(-x^3 + 3x^2 - 1\right)$

$y = -\ln\left(-x^3 + 3x^2 - 1\right)$

Note: This solution is valid on an interval containing $x = 1$ for which $-x^3 + 3x^2 - 1 > 0$.

$$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

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6A, 1

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6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

$$\frac{dy}{dx} = \cancel{e}^0(3-6)$$

$$\frac{dy}{dx} = -3$$

$$y - 0 = -3(x-1)$$

$$\boxed{y = -3x + 3}$$

$$y(1.2) = -3(1.2) + 3$$

$$\boxed{y = -.6}$$

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6A$_2$

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

$$\int \frac{1}{e^y}\, dy = \int 3x^2 - 6x \, dx$$

e^{-y}

$$-e^{-y} = x^3 - 3x^2 + C$$

$$e^{-y} = -x^3 + 3x^2 + C$$

$$-y = \ln(-x^3 + 3x^2 + C)$$

$$y = -\ln(-x^3 + 3x^2 + C)$$

$$0 = -\ln(-(1)^3 + 3 + C)$$

$$0 = -\ln(-1 + 3 + C)$$

$$0 = -\ln(2 + C)$$

$$2 + C = 1$$

$$C = -1$$

$$\boxed{y = -\ln(-x^3 + 3x^2 - 1)}$$

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6B,

6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

$mT = e^y(3x^2 - 6x)$, $(1,0)$

$= e^0(3(1)^2 - 6(1))$

$= -3$

$y - y_1 = mT(x - x_1)$

$y - 0 = -3(x - 3)$

$y = -3x + 9$

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6B$_2$

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

$dy = e^y(3x^2-6x)dx$

$\frac{dy}{e^y} = (3x^2-6x)dx$

$\int e^{-y}dy = \int (3x^2-6x)dx$

$-e^{-y} = x^3-3x^2+C, (1,0)$

$-e^{-0} = (1)^3-3(1)^2+C$

$1 = 1-3+C$

$C = 3$

$-e^{-y} = x^3-3x^2+3$

$e^{-y} = -x^3+3x^2-3$

$-y = \ln(-x^3+3x-3)$

$y = -\ln(-x^3+3x-3)$

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6C1

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6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

$\frac{dy}{dx} = e^0(3(1)^2 - 6(1))$

$1(3-6)$

$y - 0 = -3(x-1)$

$\boxed{y = -3x + 3}$

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6C2

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

$$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = e^y(3x^2-6x)\,dx$$

$$\frac{dy}{e^y} = \frac{\cancel{e^y}(3x^2-6x)\,dx}{\cancel{e^y}}$$

$$\int \frac{1}{e^y}\,dy = \int 3x^2-6x\,dx$$

$$\frac{2x^3}{3} - 3x^2$$

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Question 6

Overview

This problem presented students with a differential equation and defined $y = f(x)$ to be the particular solution to the differential equation passing through a given point. Part (a) asked students to write an equation for the line tangent to the graph of f at the given point, and then to use this tangent line to approximate $f(x)$ at a nearby value of x. Students needed to recognize that the slope of the tangent line is the value of the derivative, given in the differential equation, at the given point. Part (b) asked for the particular solution to the differential equation that passes through the given point. Students should have used the method of separation of variables to solve the differential equation.

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 1 point in part (a) and 5 points in part (b). In part (a) the student correctly computes the slope using the given derivative and earned the first point. The student gives a line through $(3, 0)$ rather than $(1, 0)$. The second point was not earned, and the student was not eligible for the third point. In part (b) the student correctly separates the differential equation and earned the first point. The student correctly antidifferentiates both the exponential function and the polynomial function, so the second and third points were earned. The student includes the constant of integration and earned the fourth point. The student uses the initial condition $(1, 0)$ and earned the fifth point. The last point was not earned because the student makes an arithmetic error while computing C.

Sample: 6C
Score: 3

The student earned 3 points: 2 points in part (a) and 1 point in part (b). In part (a) the student correctly computes the slope using the given derivative and earned the first point. The student gives a correct tangent line and earned the second point. The student does not use the tangent line to approximate $f(1.2)$, so the third point was not earned. In part (b) the student correctly separates the differential equation and earned the first point. The student does not antidifferentiate the exponential or the polynomial correctly. The second and third points were not earned. The student did not earn any additional points in this part.