Question 1

Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

(a) $\dfrac{A(30) - A(0)}{30 - 0} = -0.197$ (or -0.196) lbs/day	1 : answer with units
(b) $A'(15) = -0.164$ (or -0.163) The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.	$2 : \begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$
(c) $A(t) = \dfrac{1}{30}\displaystyle\int_0^{30} A(t)\,dt \Rightarrow t = 12.415$ (or 12.414)	$2 : \begin{cases} 1 : \dfrac{1}{30}\displaystyle\int_0^{30} A(t)\,dt \\ 1 : \text{answer} \end{cases}$
(d) $L(t) = A(30) + A'(30) \cdot (t - 30)$ $A'(30) = -0.055976$ $A(30) = 0.782928$ $L(t) = 0.5 \Rightarrow t = 35.054$	$4 : \begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

$$\frac{A(30)-A(0)}{30-0} = \frac{-5.904}{30} \approx -.197 \ \frac{\text{pounds}}{\text{day}}$$

Do not write beyond this border.

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

$$A'(t) \approx -.478(.931)^t$$

$$A'(15) \approx -.164 \ \frac{\text{pounds}}{\text{day}}$$

The amount of grass clippings in the bin is decreasing (decomposing) at a rate of .164 pounds per day at time = 15 days

Unauthorized copying or reuse of any part of this page is illegal.

Continue problem 1 on page 5.

 1A2

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

average amount $= \frac{1}{30}\int_0^{30} A(t)\,dt \approx 2.75263511$

$A(t) = 2.75263511 = 6.687(.931)^t$

this occurs at $t \approx 12.414$ days

Do not write beyond this border. Do not write beyond this border.

(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

$L(t)$ is the tangent line to $A(t)$ at $t=30$

$A(30) \approx .783 \Rightarrow (30, .783) = (t, A(t))$

$A'(30) \approx -.056$ let $-.056 = M$

$(y-y_1) = M(x-x_1)$ so for this problem,

$(A(t) - .783) = -.056(t-30)$

When there are .5 pounds of grass, $A(t) = .5$,

$(.5 - .783) = -.056(t-30)$

$t = 35.054$ days

GO ON TO THE NEXT PAGE.

 1B1

Do not write beyond this border.

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

$$\frac{A(30)-A(0)}{30-0} = \frac{.7829178 - 6.687}{30} = -.196 \frac{\text{pounds}}{\text{day}}$$

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

$$A'(t) = -.47809\,(.931)^x \big|_{x=15} = -.163 \frac{\text{pounds}^2}{\text{day}}$$

this value represents the rate of which the grass is decomposing in pound² per day @ $t=15$ days

Continue problem 1 on page 5.

1B₂

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

$$\frac{1}{30-0}\int_0^{30} A(t)\,dt = 2.752$$

$$2.752 = 6.687(.931)^t$$

$$t = 12.418$$

(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

$$A(30) = .783$$

$$(30, .783)$$

$$A'(30) = -.47809(.931)^{30} = -.056$$

$$L(t) - .783 = -.056(t-30)$$

$$L(t) = -.056t + 1.68 + .783$$

$$L(t) = -.056t + 2.463$$

$$.5 = -.056t + 2.463$$

$$t = 35.053$$

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $\overset{\text{in bin}}{A(t)} = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

$a^u = \ln a \cdot a^u \cdot u'$

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

$\frac{1}{30-0}\int_0^{30} A'(t)\,dt = \frac{1}{30} \cdot 6.687 \cdot \int_0^{30} [(\ln .931)(.931^t)]\,dt$

$= -.1968024044$ lbs of grass/day

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

$A'(t) = 6.687 \cdot \ln(.931) \cdot (.931^t)$

$A'(15) = 6.687 \cdot \ln(.931) \cdot (.931^{15}) = -.1635905804$

the rate at which the amount of grass in the bin is decomposing is $-.1635905805$ pounds per day

Do not write beyond this border.

Continue problem 1 on page 5.

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

$$A(t) = \frac{1}{30}\int_0^{30} A(t)\,dt$$

$$6.687(.931)^t = \frac{1}{30}\left[(6.687(.931)^{30}) - (6.687(.931)^{0})\right]$$

$$6.687(.931)^t = 24981.18241$$

there is not t

Do not write beyond this border.

(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

$$.5 = L(t)$$

GO ON TO THE NEXT PAGE.

Question 1

Overview

In this problem students were given $A(t)$, a model for the amount of grass clippings, in pounds, contained in a bin at time t days for $0 \le t \le 30$. In part (a) students were asked to show the calculation of the average rate of change of $A(t)$ over the interval $0 \le t \le 30$ and specify the units of the result – pounds per day. In part (b) students were asked to calculate the derivative of $A(t)$ at $t = 15$, either by using the calculator or by applying basic derivative formulas to $A(t)$ to obtain $A'(t)$ and then evaluating $A'(t)$ at $t = 15$. This answer is negative. Therefore, students needed to interpret the absolute value of this answer as the rate at which the amount of grass clippings in the bin is decreasing, in pounds per day, at time $t = 15$ days. In part (c) students were given two tasks. First, students needed to set up and evaluate the integral expression for the average value of $A(t)$ over the interval $0 \le t \le 30$, namely $\frac{1}{30}\int_0^{30} A(t)\,dt$. Second, students needed to set up and solve the equation $A(t) = \frac{1}{30}\int_0^{30} A(t)\,dt$ for t in the interval $0 \le t \le 30$. In part (d) students needed to compute $A(30)$, $A'(30)$, and write $L(t) = A(30) + A'(30)(t - 30)$. Students were to then solve the equation $L(t) = 0.5$.

Sample: 1A
Score: 9

The student earned all 9 points. In part (d) the student evidently stored more accurate intermediate values in the calculator because the correct answer is presented.

Sample: 1B
Score: 6

The student earned 6 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and 3 points in part (d). In part (a) the student's work is correct. In part (b) the student earned the first point for $A'(15)$. The units are incorrect, so the interpretation point was not earned. The student's description seems to suggest that the student thinks that $A'(15)$ is a rate of a rate or that $A'(15)$ suggests that A is decreasing at a negative rate at $t = 15$. In part (c) the student earned the first point for $\frac{1}{30-0}\int_0^{30} A(t)\,dt$. The student's answer is not accurate to three decimal places. In part (d) the student earned the first two points for $L(t)$ and the point for setting $L(t) = 0.5$. The student's answer is not accurate to three decimal places.

Question 1 (continued)

Sample: 1C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student earned the first point for $A'(15)$. The interpretation point was not earned because the student's claim that the grass clippings decompose at a negative rate is incorrect. The grass clippings are decreasing at the rate of $|A'(15)|$ pounds per day at $t = 15$. In part (c) the student earned the first point for $\frac{1}{30}\int_0^{30} A(t)\,dt$. In part (d) the student is not eligible for the point for setting $L(t) = 0.5$ because neither of the first 2 points was earned.