AP® CALCULUS AB/CALCULUS BC
2015 SCORING GUIDELINES

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t)\,dt = 76.570$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$

Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

$2 : \begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t, $0 \le t \le 8$, is $30 + \int_0^t [R(x) - D(x)]\,dx.$

$R(t) - D(t) = 0 \Rightarrow t = 0,\ 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

$3 : \begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $30 + \int_0^w [R(t) - D(t)]\,dt = 50$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially ~~blocked, allowing water to drain~~ out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

$$\int_0^8 R(t)\,dt = \int_0^8 20 \sin \frac{t^2}{35}\,dt = \boxed{76.570 \text{ ft}^3}$$

During the eight hour interval, about 76.570 cubic feet of water flow into the drainpipe

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

Total water: $T(x)$ $\quad T'(x) = R(x) - D(x) = 20 \sin \frac{t^2}{35} + .04t^3 - .4t^2 - .96t$

$T'(3) = 20 \sin 9/35 + .04(27) - .4(9) - .96(3) = -.314 < 0$

after three hours, the amount of water in the pipe is $\boxed{\text{decreasing}}$ because the derivative of the amount of water (the difference between water entering and leaving) is less than zero at 3 hours.

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(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

$T'(t) = 0$ @ $t = 0, 3.2716584$

$T(t) = T(0) + \int_0^t T'(t)\,dt = 30 + \int_0^t T'(t)\,dt$

$T(0) = 30$

$T(3.272) = 27.965$

$T(8) = 48.544$

after testing all critical numbers and endpoints for their values, the amount of water in the pipe achieves a minimum value of about 27.965 after about $\boxed{3.272 \text{ hours}}$

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

$50 = T(w)$

$50 = 30 + \int_0^w T'(t)\,dt$

$\boxed{20 = \int_0^w T'(t)\,dt} = \int_0^w [R(t) - D(t)]\,dt$

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1&2
1B

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1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

$$\int_0^8 20\sin\left(\frac{t^2}{35}\right) dt = 76.57035295 \text{ ft}^3$$

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

At $t = 3$ hours, $R(t) < D(t)$ $\therefore$ The amount of water in the pipe is decreasing

$$R(3) = 20\sin\left(\frac{3^2}{35}\right) = 5.086 \text{ ft}^3/\text{hour}$$

$$D(3) = -.04(3)^3 + .4(3)^2 + 0.96(3) = 5.4 \text{ ft}^3/\text{hour}$$

Continue problem 1 on page 5.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

$R(t) = 0 = 20\sin\left(\frac{t^2}{35}\right)$ @ $t = 0,\ 10.486$

+		+		+
	0		10.486	

abs minimum @ $t = 0$

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

$$50 = 30 + \int_0^w 20\sin\left(\frac{t^2}{35}\right)dt - \int_0^w -.04t^3 + .4t^2 + .96t\,dt$$

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1 of 2
1C

1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

$(0, 30)$

$R(8) = 20\sin\left(\frac{8^2}{35}\right)$

$= 20\sin\left(\frac{64}{35}\right)$

$\boxed{R(8) = 19.3392}$

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

$R(3) = 20\sin\left(\frac{3^2}{35}\right)$

$= 20\sin\left(\frac{9}{35}\right)$

$R(3) = 5.08637$

$D(t) = -0.04(3)^3 + 0.4(3)^2 + 0.96(3)$

$D(t) = 5.4$

The amount of water in the pipe is decreasing at time $t = 3$ hours, because the rate at which the water is draining out the other end of the pipe is greater, $D(3) = 5.4$, than the rate at which the rainwater flows into the drainpipe, $R(3) = 5.08637$.

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1C

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(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

$$20\sin\left(\frac{t^2}{35}\right) = -0.04t^3 + 0.4t^2 + 0.96t$$

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

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Question 1

Overview

In this problem students were given $R(t)$, the rate of flow of rainwater into a drainpipe, in cubic feet per hour, and $D(t)$, the rate of flow of water out of the pipe, in cubic feet per hour. Both $R(t)$ and $D(t)$ are defined on the time interval $0 \le t \le 8$. The amount of water in the pipe at time $t = 0$ is also given. In part (a) students needed to use the definite integral to compute the amount of rainwater that flows into the pipe during the interval $0 \le t \le 8$. Students had to set up the definite integral $\int_0^8 R(t)\,dt$ and evaluate the integral using the calculator. In part (b) students should have recognized that the rate of change of the amount of water in the pipe at time t is given by $R(t) - D(t)$. Students were expected to calculate $R(3) - D(3)$ using the calculator and find that the result is negative. Therefore, the amount of water in the pipe is decreasing at time $t = 3$. In part (c) students had to find the time t, $0 \le t \le 8$, at which the amount of water in the pipe is at a minimum. Students were expected to set up an integral expression such as $30 + \int_0^t [R(x) - D(x)]\,dx$ for the amount of water in the pipe at time t. Students should have realized that an absolute minimum exists since they are working with a continuous function on a closed interval, and this minimum must occur at either a critical point or at an endpoint of the interval. Students were expected to use the calculator to solve $R(t) - D(t) = 0$ and find the single critical point at $t = 3.272$ on the interval $0 < t < 8$. Students should have stored the full value for t in the calculator and used the calculator to evaluate the function at the critical point and the endpoints. In this case the amount of water is at a minimum at the single critical point. In part (d) students were asked to write an equation involving one or more integrals that gives the time w when the pipe will begin to overflow. Students were expected to set up an equation using the initial condition, an integral expression, and the holding capacity of the pipe, such as

$30 + \int_0^w [R(t) - D(t)]\,dt = 50.$

Sample: 1A
Score: 9

The response earned all 9 points.

Sample: 1B
Score: 6

The response earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In parts (a) and (b), the student's work is correct. In part (c) the student works with $R(t)$ rather than $R(t) - D(t)$. In part (d) the student's work is correct.

Sample: 1C
Score: 3

The response earned 3 points: no points in part (a), 2 points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student finds the rate at which water enters the pipe rather than the total amount. In part (b) the student's work is correct. In part (c) the student earned the first point for considering $R(t) - D(t) = 0$.