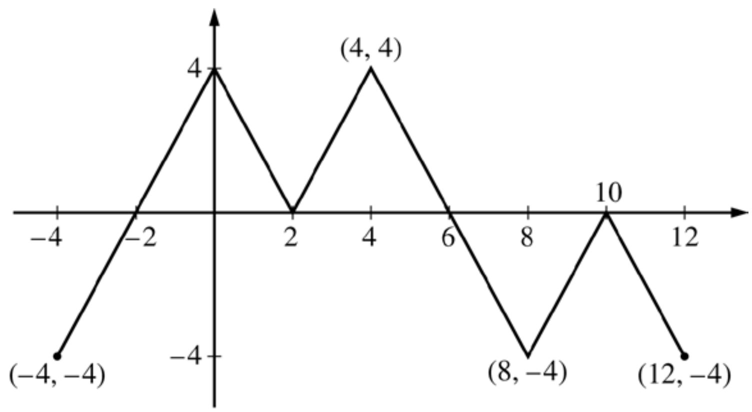
## Question 3

The figure above shows the graph of the piecewise-linear function $f$. For $-4 \le x \le 12$, the function $g$ is defined by $g(x) = \int_2^x f(t)\,dt.$




Graph of $f$

(a) Does $g$ have a relative minimum, a relative maximum, or neither at $x = 10$ ? Justify your answer.

(b) Does the graph of $g$ have a point of inflection at $x = 4$ ? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of $g$ on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

---

1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

(a) The function $g$ has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \le 0$ for $8 \le x \le 12$.

1 : answer with justification

(b) The graph of $g$ has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \le x \le 4$ and decreasing for $4 \le x \le 8$.

1 : answer with justification

(c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

| $x$ | $g(x)$ |
|---|---|
| −4 | −4 |
| −2 | −8 |
| 6 | 8 |
| 12 | −4 |

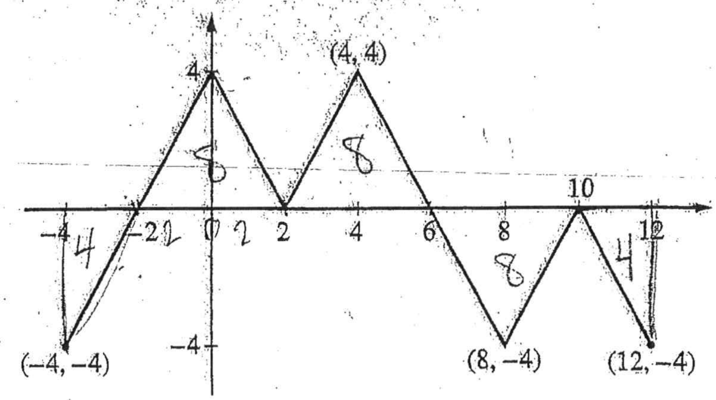
On the interval $-4 \le x \le 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

$$4 : \begin{cases} 1 : \text{considers } x = -2 \text{ and } x = 6 \\ \quad \text{as candidates} \\ 1 : \text{considers } x = -4 \text{ and } x = 12 \\ 2 : \text{answers with justification} \end{cases}$$

(d) $g(x) \le 0$ for $-4 \le x \le 2$ and $10 \le x \le 12$.

2 : intervals

**NO CALCULATOR ALLOWED**




Graph of $f$

3. The figure above shows the graph of the piecewise-linear function $f$. For $-4 \le x \le 12$, the function $g$ is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does $g$ have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

$g'(x) = f(x)$

g does not have a relotive minimum or maximum at $x=10$ because $g'(x) = f(x)$ does not change sign at this point

(b) Does the graph of $g$ have a point of inflection at $x = 4$? Justify your answer.

$g''(x) = f'(x)$

$f'(x) = g''(x)$ does change sign at $x=4$ so g does have a point of inflection at this point



Continue problem 3 on page 15.

(c) Find the absolute minimum value and the absolute maximum value of $g$ on the interval $-4 \le x \le 12$. Justify your answers.

$g'(x) = f(x) = 0 \quad x = -2 \quad x = 2$
$x = 6 \quad x = 10$

does not change sign at $x = 2$ and $x = 10$

| $x$ | $g(x)$ |
|---|---|
| $-4$ | $\int_2^{-4} f(t)\,dt = -8 + 4 = -4$ |
| $-2$ | $\int_2^{-2} f(t)\,dt = -8$ |
| $6$ | $\int_2^{6} f(t)\,dt = 8$ |
| $12$ | $\int_2^{12} f(t)\,dt = 8 - 8 - 4 = -4$ |

The absolute minimum value of $g$ on the interval $-4 \le x \le 12$ is $-8$ and the absolute maximum value of $g$ is $8$

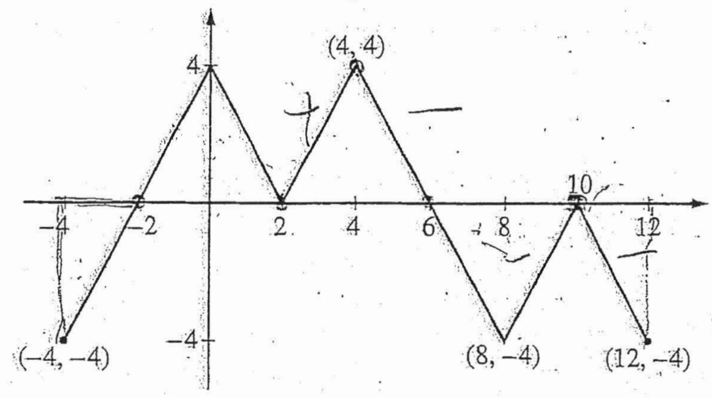
(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

$g(x) = \int_2^{x} f(t)\,dt \le 0$

$g(x) = 0$ at $x = 2$ and $x = 10$

$g(x) \le 0$ in the intervals $-4 \le x \le 2$ and $10 \le x \le 12$

**NO CALCULATOR ALLOWED**




Graph of $f$

3. The figure above shows the graph of the piecewise-linear function $f$. For $-4 \le x \le 12$, the function $g$ is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does $g$ have a relative minimum, a relative maximum, or neither at $x = 10$ ? Justify your answer.

$g'(x) = f(x)$

$g'(10) = f(10) = 0$

g is neither at $x=10$ bc $g'(x)$ does not change from pos to neg or neg to pos at $x = 10$.

(b) Does the graph of $g$ have a point of inflection at $x = 4$ ? Justify your answer.

$g''(x) = f'(x)$

$f'(4) = 0$

g has a poi at $x=4$ bc $g''(4) = 0$ and $g''(x) > 0$ when $2 \le x < 4$ and $g''(x) < 0$ when $4 < x < 8$.

Do not write beyond this border.



Continue problem 3 on page 15.

(c) Find the absolute minimum value and the absolute maximum value of $g$ on the interval $-4 \le x \le 12$. Justify your answers.

$g'(x) = f(x) = 0$

$x = -2, 2, 6, 10$

| $x$ | $g(x)$ |
|---|---|
| $-4$ | $-\int_{-4}^{2} f(t)\,dt = -\left[(\frac{1}{2})(-4)(2) + (\frac{1}{2})(4)(4)\right] = -(-4+8) = -4$ |
| $-2$ | $-\int_{-2}^{2} f(t)\,dt = -\left[(\frac{1}{2})(4)(4)\right] = -8$ |
| $2$ | $\int_{2}^{2} f(t)\,dt = 0$ |
| $6$ | $\int_{2}^{6} f(t)\,dt = (\frac{1}{2})(4)(4) = 8$ |
| $10$ | $\int_{2}^{10} f(t)\,dt = 0$ |
| $12$ | $\int_{2}^{12} f(t)\,dt = (\frac{1}{2})(-4)(2) = -4$ |

abs max $\rightarrow x = 6$ abs min $\Rightarrow x = -2$

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

$\int_{2}^{x} f(t)\,dt \le 0$

$\boxed{\begin{matrix}(10, 12) \\ (-4, 2)\end{matrix}}$

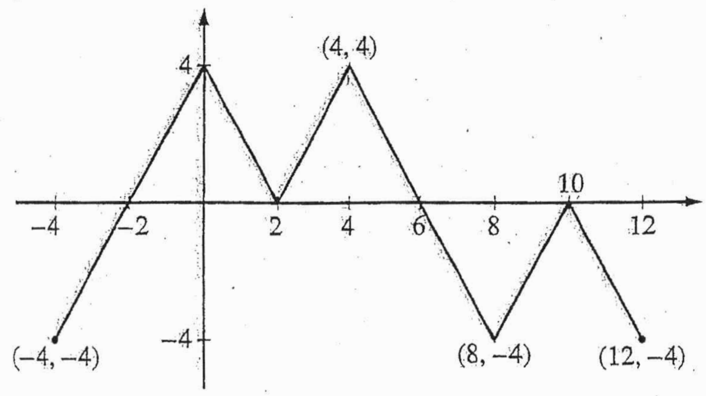
Do not write beyond this border.

Do not write beyond this border.



GO ON TO THE NEXT PAGE.

**NO CALCULATOR ALLOWED**




Graph of $f$

3. The figure above shows the graph of the piecewise-linear function $f$. For $-4 \le x \le 12$, the function $g$ is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does $g$ have a relative minimum, a relative maximum, or neither at $x = 10$ ? Justify your answer.

$g'(x) = f(x)$

Since $g'(x) = f(x)$, the graph of $g$ has a relative maximum at $x=10$ because the graph of $f$ increases before $x = 10$ and decreases after $x=10$ and $x=10$ is a critical point.

(b) Does the graph of $g$ have a point of inflection at $x = 4$ ? Justify your answer.

$g''(x) = f'(x)$

Since $g''(x) = f'(x)$, the graph of $g$ has an inflection point at $x=4$ because the graph of $f$ increases before $x=4$ and decreases after $x=4$.



Continue problem 3 on page 15.

**NO CALCULATOR ALLOWED**

(c) Find the absolute minimum value and the absolute maximum value of $g$ on the interval $-4 \le x \le 12$. Justify your answers.

$g'(x) = f(x) = 0$

$x = -2, 2, 6, 10$

absolute maximum at $x = 10$ and absolute minimum at $x = 2$

. The absolute value for both extremas are 0 since

It is found by $g'(x) = f(x) = 0$.

Do not write beyond this border.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

$g(x)$ is decreasing when $g'(x)$ is $\le 0$ and $g''(x)$ is $\le 0$

since $g'(x) = f(x)$ and $g''(x) = f'(x)$, we know that

$6 < x < 10$ and $10 < x < 12$ are the only intervals

where both $g(x)$ and $g''(x)$, which is $f(x)$ and $f'(x)$,

are decreasing (having the same sign).



GO ON TO THE NEXT PAGE.

## Question 3

### Overview

In this problem students were given the graph of $f$, a piecewise-linear function defined on the interval $[-4, 12]$. A second function $g$ is defined by $g(x) = \int_2^x f(t)\, dt$. In part (a) students needed to determine whether $g$ has a relative minimum, a relative maximum, or neither at $x = 10$, and justify their answer. Using the Fundamental Theorem of Calculus, students needed to recognize that $g'(x) = f(x)$ for all $x$ in the interval $[-4, 12]$. Since $g'(10) = f(10) = 0$ and $f(x) \le 0$ for $[8, 12]$, the First Derivative Test may be applied to conclude that there is no relative extremum at $x = 10$. In part (b) students needed to determine whether the graph of $g$ has a point of inflection at $x = 4$, and justify their answer. Since $g'(x) = f(x)$, the graph of $g$ has a point of inflection at $x = 4$ because $f$ changes from increasing to decreasing at $x = 4$. In part (c) students needed to find the absolute minimum value and the absolute maximum value of $g$ on $[-4, 12]$. Since $g'(x) = f(x)$, students were expected to find relative extrema of $g$ by identifying $x$-values where $f$ changes sign. The absolute extrema occur either at the endpoints of the interval or at the relative extrema. By comparing the values of $g$ at the four candidate $x$-values, students choose and justify the absolute extrema. Properties of the definite integral and the relation of the definite integral to accumulated area must be used to find the values of $g$. In part (d) students needed to find all intervals in $[-4, 12]$ for which $g(x) \le 0$. This part also required properties of the definite integral and the relation of the definite integral to accumulated area.

**Sample: 3A**
**Score: 9**

The response earned all 9 points. The student earned the $g'(x) = f(x)$ point in part (a). In part (a) the student earned the point with justification "$g'(x) = f(x)$ does not change sign at this point." In part (b) the student earned the point with justification "$f'(x) = g''(x)$ does change sign at $x = 4$." In part (c) the student identifies the absolute minimum and absolute maximum values with a candidates test that uses the necessary critical points. In part (d) the student gives the two correct closed intervals.

**Sample: 3B**
**Score: 6**

The response earned 6 points: 1 point for $g'(x) = f(x)$, 1 point in part (a), no points in part (b), 3 points in part (c), and 1 point in part (d). The student earned the $g'(x) = f(x)$ point in part (a). In part (a) the student earned the point with justification "$g'(x)$ does not change from pos to neg or neg to pos at $x = 10$." In part (b) the student gives the correct answer but includes an incorrect statement that $g''(4) = 0$. In part (c) the student earned the first 2 points. The student does not identify the absolute minimum as $-8$ or the absolute maximum as 8. The student earned 1 of the 2 answers with justification points. In part (d) the student does not include the endpoints of the intervals, so 1 point was earned.

**Sample: 3C**
**Score: 3**

The response earned 3 points: 1 point for $g'(x) = f(x)$, no points in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). The student earned the $g'(x) = f(x)$ point in part (a). In part (a) the student has

### Question 3 (continued)

an incorrect answer. In part (b) the student's work is correct. In part (c) the student earned the first point by identifying $x = -2$ and $x = 6$ in the second line. The student earned no other points. In part (d) the student has an incorrect interval $(6, 10)$ that has no values where $g(x) \le 0$.