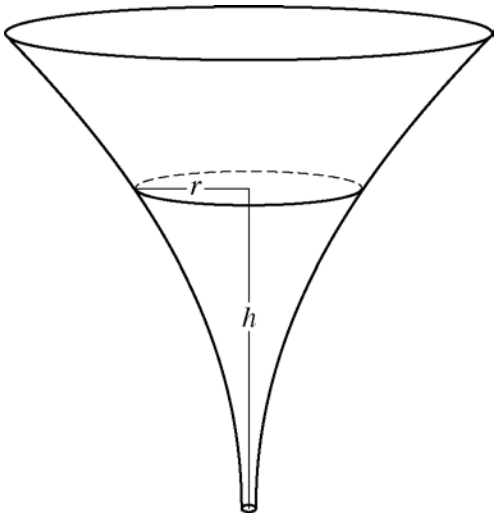
AP® CALCULUS AB/CALCULUS BC
2016 SCORING GUIDELINES

Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

(a) Average radius $= \frac{1}{10}\int_0^{10} \frac{1}{20}\left(3 + h^2\right)\,dh = \frac{1}{200}\left[3h + \frac{h^3}{3}\right]_0^{10}$

$= \frac{1}{200}\left(\left(30 + \frac{1000}{3}\right) - 0\right) = \frac{109}{60}$ in

3 : { 1 : integral; 1 : antiderivative; 1 : answer }

(b) Volume $= \pi\int_0^{10} \left(\left(\frac{1}{20}\right)\left(3 + h^2\right)\right)^2\,dh = \frac{\pi}{400}\int_0^{10}\left(9 + 6h^2 + h^4\right)\,dh$

$= \frac{\pi}{400}\left[9h + 2h^3 + \frac{h^5}{5}\right]_0^{10}$

$= \frac{\pi}{400}\left(\left(90 + 2000 + \frac{100000}{5}\right) - 0\right) = \frac{2209\pi}{40}$ in^3

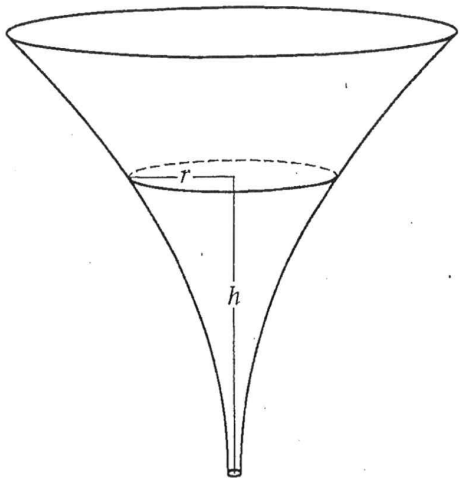
3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(c) $\frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt}$

$-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{5}\cdot\frac{10}{3} = -\frac{2}{3}$ in/sec

3 : { 2 : chain rule; 1 : answer }



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

$$r_{avg} = \frac{1}{200}\int_0^{10}\left(3+h^2\right)dh$$

$$= \frac{1}{200}\left[3h + \frac{h^3}{3}\right]_0^{10}$$

$$= \frac{1}{200}\left(30 + \frac{1000}{3}\right)$$

$$= \frac{109}{60} \text{ inches.}$$

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Continue problem 5 on page 19.

(b) Find the volume of the funnel.

$$V = \frac{\pi}{400}\int_0^{10}(3+h^2)^2\,dh$$
$$= \frac{\pi}{400}\int_0^{10}(9+6h^2+h^4)\,dh$$
$$= \frac{\pi}{400}\left(9h+2h^3+\frac{h^5}{5}\right)\Big|_0^{10}$$
$$= \frac{\pi}{400}(90+2000+20000)$$
$$= \frac{2209\pi}{40}\ in^3$$

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

$$r = \frac{1}{20}(3+h^2)$$
$$\frac{dr}{dt} = \left(\frac{h}{10}\right)\frac{dh}{dt}$$
$$-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt}$$
$$\frac{dh}{dt} = -\frac{2}{3}\ \text{inch per second}$$

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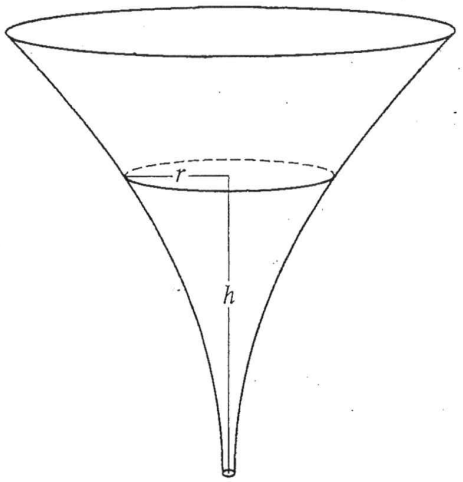
NO CALCULATOR ALLOWED

5B



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

$$\frac{1}{10}\int_0^{10} \frac{1}{20}\left(3+h^2\right)dh = \frac{1}{200}\int_0^{10}\left(3+h^2\right)dh$$

$$= \frac{1}{200}\left(3h + \frac{1}{3}h^3\right) + C\Big]_0^{10}$$

$$= \frac{1}{200}\left(\left(30 + \frac{1000}{3}\right) - (0)\right) \text{ inches}$$

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(b) Find the volume of the funnel.

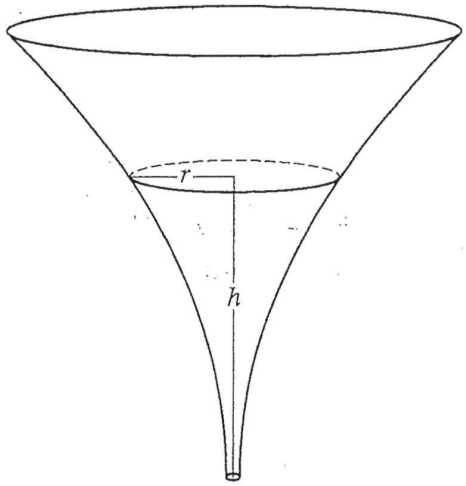
$$\pi \int_0^{10} \frac{1}{20}(3+h^2)\,dh = V$$

$$\frac{\pi}{20}\left(3h + \frac{1}{3}h^3\right) + C \Big]_0^{10} = \frac{\pi}{20}\left(30 + \frac{1000}{3}\right) \text{ cubic inches}$$

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

$$\frac{dr}{dt} = \frac{1}{20}(2h)\left(\frac{dh}{dt}\right)$$

$$-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt} \qquad \frac{dh}{dt} = \frac{-10}{5(3)} \text{ inches per second}$$



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(a) Find the average value of the radius of the funnel.

$r = \frac{1}{20}(3+h^2)$ where $0 \le h \le 10$.

$$\overline{r} = \frac{r(10) - r(0)}{10-0} = \frac{\frac{103}{20} - \frac{3}{20}}{10} = \frac{1}{2} \text{ (inches)}.$$

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Continue problem 5 on page 19.

(b) Find the volume of the funnel.

$V = \int_0^{10} \pi r^2 \, dh = \pi \int_0^{10} \left(\frac{3+h^2}{20}\right)^2 dh$

$= \pi \frac{1}{6h}\left(\frac{3+h^2}{20}\right)^3 \Big]_0^{10}$

$= \pi \cdot \frac{1}{60}\left(\frac{103}{20}\right)^2 - 0$

$= \frac{\pi}{60} \cdot \frac{103^2}{400}$

$= \frac{103^2 \pi}{24000} \text{ (inches}^3\text{)}$

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

$r = \frac{3}{20} + \frac{h^2}{20}$

$r' = \frac{h}{10} \cdot \frac{dh}{dt} = -\frac{1}{5}$

$h = 3$

$r' = \frac{3}{10} \cdot \frac{dh}{dt} = -\frac{1}{5}$

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AP® CALCULUS AB/CALCULUS BC
2016 SCORING COMMENTARY

Question 5

Overview

In this problem students were presented with a funnel of height 10 inches and circular cross sections. At height h the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where r and h are measured in inches. In part (a) students needed to find the average value of the radius of the funnel. This required evaluating $\frac{1}{10}\int_0^{10} \frac{1}{20}\left(3 + h^2\right)\, dh$ by finding an antiderivative. In part (b) students needed to find the volume of the funnel. By incorporating the fact that the cross sections are circular, the students were expected to set up and evaluate an integral of the form $\pi\int_0^{10} r^2\, dh = \pi\int_0^{10} \left(\left(\frac{1}{20}\right)\left(3 + h^2\right)\right)^2\, dh$. In part (c) students were given that the funnel contains liquid that is draining from the bottom. When the height of the liquid is 3 inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ in/sec. Students were expected to find the rate at which the height is changing at this instant. To solve this related rates problem, students needed to use $r = \frac{1}{20}\left(3 + h^2\right)$ and take the derivative with respect to t.

Sample: 5A
Score: 9

The response earned all 9 points.

Sample: 5B
Score: 6

The response earned 6 points: 3 points in part (a), no points in part (b), and 3 points in part (c). In part (a) the student's work is correct. In part (b) the student does not present a correct integrand in the integral for volume and did not earn the first point. Without a correct integrand, the student was not eligible for the other points. In part (c) the student's work is correct.

Sample: 5C
Score: 3

The response earned 3 points: no points in part (a), 1 point in part (b), and 2 points in part (c). In part (a) the student does not present an integral and did not earn the first point. Without presentation of an integral, the student was not eligible for the other points. In part (b) the student presents a correct integrand in the integral for volume and earned the first point. The student does not antidifferentiate correctly and did not earn the second point. The student was not eligible for the third point. In part (c) the student uses the chain rule correctly to find an equation relating $\frac{dh}{dt}$ to $\frac{dr}{dt}$ and earned the first 2 points. The student does not solve for $\frac{dh}{dt}$ and did not earn the third point.