2018

AP® CollegeBoard

AP Calculus AB

Sample Student Responses and Scoring Commentary

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Question 1

(a) $\int_0^{300} r(t)\,dt = 270$ According to the model, 270 people enter the line for the escalator during the time interval $0 \le t \le 300$.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) $20 + \int_0^{300} (r(t) - 0.7)\,dt = 20 + \int_0^{300} r(t)\,dt - 0.7 \cdot 300 = 80$ According to the model, 80 people are in line at time $t = 300$.	$2 : \begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$
(c) Based on part (b), the number of people in line at time $t = 300$ is 80. The first time t that there are no people in line is $300 + \frac{80}{0.7} = 414.286$ (or 414.285) seconds.	1 : answer
(d) The total number of people in line at time t, $0 \le t \le 300$, is modeled by $20 + \int_0^t r(x)\,dx - 0.7t$. $r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$	$4 : \begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

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1. People enter a line for an escalator at a rate modeled by the function r given by q

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$$\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt = \boxed{270}$$

(b) qDuring the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$.7(300) = 210$

$20 + 270 = 290$

$290 - 210 = \boxed{80}$

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(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

$(t-300)(.7) - 80 = 0$

$.7t - 210 - 80 = 0$

$.7t = +290$

$\boxed{t = 414.286s}$

(d) JFor $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

p= total people

$\frac{dp}{dt} = r(t) - .7$

$0 = r(t) - .7$

$t = 166.575$

$t = 33.013$

$p(t) = \int_0^t r(\cancel{x}) - .7 \, dx + 20$

t	p(t)
0	20
33.013	3.803
166.575	158.07014
300	80

minimum at time t=33.013s when 4 people are in line

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1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

$S(0)=20$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$? v

$\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt =$ 270 people enter the line for the escalator during the time interval $0 \le t \le 300$

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$L(t) = 0.7$

$20 + \int_0^{300} r(t)\,dt - \int_0^{300} 0.7\,dt$

$20 + 270 - 210$

At time $t = 300$ seconds, there are 80 people in line for the escalator.

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(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

For $t > 300$, the first time t that there are no people in line for the escalator is $t = 325$

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

$44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 - 0.7$

$44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 = 0.7$

$20 + \int_0^{33.013298 \to A} r(t)\,dt - \int_0^A 0.7\,dt$

At time $t = 33.013$ seconds there is a minimam of 3.803 people on the escalator.

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1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$r(t) = 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 \quad (0, 300)$

$\text{people} = \int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 dt$

$\text{people} = \boxed{56700 \text{ people}}$

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$20 + \int_0^{300} r(t)\, dt - \left(\int_0^{300} (0.7\, dt)\right)$

$\left[20 + \int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 dt\right] - \int_0^{300} .7\, dt$

$= \boxed{80 \text{ people}}$

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(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

$r(t) = 0$ $t > 300$

There are no people in line for the escalator first at time $t = 300$.

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

rate enter − rate exit = 0

* $r(t) = 0$ and changes inc → dec

$r(t) = 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 = 0$

$t = 150$ sec.

Question 1

Overview

The context of this problem is a line of people waiting to get on an escalator. The function r models the rate at which people enter the line, where $r(t) = 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7$ for $0 \le t \le 300,$ and $r(t) = 0$ for $t > 300;$ $r(t)$ is measured in people per second, and t is measured in seconds. Further, it is given that people exit the line to get on the escalator at a constant rate of 0.7 person per second and that there are 20 people in the line at time $t = 0.$ In part (a) students were asked how many people enter the line for the escalator during the time interval $0 \le t \le 300.$ A correct response demonstrates the understanding that the number of people entering the line during this time interval is obtained by integrating the rate at which people enter the line across the time interval. Thus, this number is the value of the definite integral $\int_0^{300} r(t)\, dt.$ A numerical value for this integral should be obtained using a graphing calculator. In part (b) students were given that there are always people in line during the time interval $0 \le t \le 300$ and were asked to determine the number of people in line at time $t = 300.$ A correct response should take into account the 20 people in line initially, the number that entered the line as determined in part (a), and the number of people that exit the line to get on the escalator. It was given in the problem statement that people exit the line at a constant rate of 0.7 person per second, so the number of people that exit the line to get on the escalator can be found by multiplying this constant rate times the duration of the interval, namely 300 seconds. In part (c) students were asked for the first time t beyond $t = 300$ when there are no people in line for the escalator. Because no more people join the line after $t = 300$ seconds, and people exit the line at the constant rate of 0.7 person per second, dividing the answer to part (b) by 0.7 gives the number of seconds beyond $t = 300$ before the line empties for the first time. Adding this quotient to 300 produces the answer. In part (d) students were asked when, during the time interval $0 \le t \le 300,$ is the number of people in line a minimum, and to determine the number of people in line (to the nearest whole number) at that time, with the added admonition to justify their answer. The Extreme Value Theorem guarantees that the number of people in line at time $t,$ given by the expression $20 + \int_0^t r(x)\, dx - 0.7t,$ attains a minimum on the interval $0 \le t \le 300.$ Correct responses should demonstrate that the rate of change of the number of people in line is given by $r(t) - 0.7.$ Solving for $r(t) - 0.7 = 0$ within the interval $0 < t < 300$ yields two critical points, t_1 and $t_2,$ so candidates for the time when the line is a minimum are $t = 0,$ $t_1,$ $t_2,$ and $t = 300.$ The number of people in line at times t_1 and t_2 is computed from $20 + \int_0^{t_1} r(x)\, dx - 0.7t_1$ and $20 + \int_0^{t_2} r(x)\, dx - 0.7t_2.$ The answer is the least of 20, these two computed values (to the nearest whole number), and the answer to part (b), together with the corresponding time t for this minimum value.

For part (a) see LO 3.3B(b)/EK 3.3B2, LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. For parts (b) and (c), see LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. For part (d) see LO 1.2B/EK 1.2B1, LO 2.3C/EK 2.3C3, LO 3.3B(b)/EK 3.3B2, LO 3.4A/EK 3.4A2, LO 3.4E/EK 3.4E1. This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, building notational fluency, and communicating.

Sample: 1A
Score: 9

The response earned all 9 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 4 points in part (d). In part (a) the response earned the first point for $\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 dt.$ The response earned the

Question 1 (continued)

second point for the answer 270. In part (b) the response earned the first point for -210 on the right side of the response area. The response earned the second point for the answer 80. In part (c) the response earned the point for the answer 414.286. What appears to be a fourth digit of 5 is not a digit but the letter s for seconds. Units are not required to earn any points in this question. In part (d) the response earned the first point for $0 = r(t) - .7$ in line 2. The response earned the second point with $t = 33.013$ in line 4. The response earned the third point for the boxed information. What appears to be a fourth digit of 5 is not a digit but the letter s for seconds. The response earned the fourth point for the candidates test demonstrated with the table. The expression for the function p, identified as "total people," supports how the values at 33.013 and 166.575 are produced.

Sample: 1B
Score: 6

The response earned 6 points: 2 points in part (a), 2 points in part (b), no point in part (c), and 2 points in part (d). In part (a) the response earned the first point for $\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7\,dt.$ The response earned the second point for the answer 270. In part (b) the response earned the first point with the term $-\int_0^{300} 0.7\,dt.$ The response earned the second point for the answer 80. In part (c) the response did not earn the point because what is presented is incorrect. In part (d) the response earned the first point in line 2 with the equation $44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 = 0.7.$ The response earned the second point with $t = 33.013$ in line 3. The response did not earn the third point for the answers because 3.803 is not rounded to a whole number. The response did not earn the fourth point because it does not have a complete justification.

Sample: 1C
Score: 3

The response earned 3 points: 1 point in part (a), 2 points in part (b), no point in part (c), and no points in part (d). In part (a) the response earned the first point for $\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7\,dt.$ The response did not earn the second point because 56700 is incorrect. In part (b) the response earned the first point with the term $-\left(\int_0^{300}(0.7\,dt)\right)$ in line 1. The response earned the second point for the answer 80. In part (c) the response did not earn the point because what is presented is incorrect. In part (d) the response did not earn the first point with any of the equations presented. The equation at the top right "rate enter $-$ rate exit $= 0$" is too formulaic and not specific to the question. The response does not identify $t = 33.013$ and did not earn the second point. As a result, the response is not eligible to earn the remaining 2 points.