

2018

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CollegeBoard

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# AP Calculus AB

## Scoring Guidelines

**AP<sup>®</sup> CALCULUS AB/CALCULUS BC  
2018 SCORING GUIDELINES**

**Question 1**

(a)  $\int_0^{300} r(t) dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval  $0 \leq t \leq 300$ .

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b)  $20 + \int_0^{300} (r(t) - 0.7) dt = 20 + \int_0^{300} r(t) dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time  $t = 300$ .

2 :  $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

(c) Based on part (b), the number of people in line at time  $t = 300$  is 80.

The first time  $t$  that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or } 414.285) \text{ seconds.}$$

1 : answer

(d) The total number of people in line at time  $t$ ,  $0 \leq t \leq 300$ , is modeled by

$$20 + \int_0^t r(x) dx - 0.7t.$$

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

4 :  $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

$t$	People in line for escalator
0	20
$t_1$	3.803
$t_2$	158.070
300	80

The number of people in line is a minimum at time  $t = 33.013$  seconds, when there are 4 people in line.

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**Question 2**

(a)  $v'(3) = -2.118$

The acceleration of the particle at time  $t = 3$  is  $-2.118$ .

1 : answer

(b)  $x(3) = x(0) + \int_0^3 v(t) dt = -5 + \int_0^3 v(t) dt = -1.760213$

The position of the particle at time  $t = 3$  is  $-1.760$ .

3 :  $\left\{ \begin{array}{l} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{array} \right.$

(c)  $\int_0^{3.5} v(t) dt = 2.844$  (or 2.843)

$$\int_0^{3.5} |v(t)| dt = 3.737$$

The integral  $\int_0^{3.5} v(t) dt$  is the displacement of the particle over the time interval  $0 \leq t \leq 3.5$ .

The integral  $\int_0^{3.5} |v(t)| dt$  is the total distance traveled by the particle over the time interval  $0 \leq t \leq 3.5$ .

3 :  $\left\{ \begin{array}{l} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t) dt \\ \text{and } \int_0^{3.5} |v(t)| dt \end{array} \right.$

(d)  $v(t) = x_2'(t)$

$$v(t) = 2t - 1 \Rightarrow t = 1.57054$$

The two particles are moving with the same velocity at time  $t = 1.571$  (or 1.570).

2 :  $\left\{ \begin{array}{l} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{array} \right.$

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**Question 3**

(a)  $f(-5) = f(1) + \int_1^{-5} g(x) dx = f(1) - \int_{-5}^1 g(x) dx$   
 $= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b)  $\int_1^6 g(x) dx = \int_1^3 g(x) dx + \int_3^6 g(x) dx$   
 $= \int_1^3 2 dx + \int_3^6 2(x-4)^2 dx$   
 $= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$

3 :  $\begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of  $f$  is increasing and concave up on  $0 < x < 1$  and  $4 < x < 6$  because  $f'(x) = g(x) > 0$  and  $f'(x) = g(x)$  is increasing on those intervals.

2 :  $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of  $f$  has a point of inflection at  $x = 4$  because  $f'(x) = g(x)$  changes from decreasing to increasing at  $x = 4$ .

2 :  $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

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**Question 4**

(a)  $H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2}$

$H'(6)$  is the rate at which the height of the tree is changing, in meters per year, at time  $t = 6$  years.

(b)  $\frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2$

Because  $H$  is differentiable on  $3 \leq t \leq 5$ ,  $H$  is continuous on  $3 \leq t \leq 5$ .

By the Mean Value Theorem, there exists a value  $c$ ,  $3 < c < 5$ , such that  $H'(c) = 2$ .

(c) The average height of the tree over the time interval  $2 \leq t \leq 10$  is given by  $\frac{1}{10 - 2} \int_2^{10} H(t) dt$ .

$$\begin{aligned} \frac{1}{8} \int_2^{10} H(t) dt &\approx \frac{1}{8} \left( \frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3 \right) \\ &= \frac{1}{8} (65.75) = \frac{263}{32} \end{aligned}$$

The average height of the tree over the time interval  $2 \leq t \leq 10$  is  $\frac{263}{32}$  meters.

(d)  $G(x) = 50 \Rightarrow x = 1$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1+x)100 - 100x \cdot 1}{(1+x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1+x)^2} \cdot \frac{dx}{dt}$$

$$\left. \frac{d}{dt}(G(x)) \right|_{x=1} = \frac{100}{(1+1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is  $\frac{3}{4}$  meter per year.

2 :  $\left\{ \begin{array}{l} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{array} \right.$

2 :  $\left\{ \begin{array}{l} 1 : \frac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using} \\ \text{Mean Value Theorem} \end{array} \right.$

2 :  $\left\{ \begin{array}{l} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{array} \right.$

3 :  $\left\{ \begin{array}{l} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{array} \right.$

Note: max 1/3 [1-0] if  
no chain rule

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**Question 5**

- (a) The average rate of change of  $f$  on the interval  $0 \leq x \leq \pi$  is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^\pi - 1}{\pi}.$$

- (b)  $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

The slope of the line tangent to the graph of  $f$  at  $x = \frac{3\pi}{2}$  is  $e^{3\pi/2}$ .

- (c)  $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

$x$	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}e^{5\pi/4}$
$2\pi$	$e^{2\pi}$

The absolute minimum value of  $f$  on  $0 \leq x \leq 2\pi$  is  $-\frac{1}{\sqrt{2}}e^{5\pi/4}$ .

- (d)  $\lim_{x \rightarrow \pi/2} f(x) = 0$

Because  $g$  is differentiable,  $g$  is continuous.

$$\lim_{x \rightarrow \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$$

By L'Hospital's Rule,

$$\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

1 : answer

2 :  $\left\{ \begin{array}{l} 1 : f'(x) \\ 1 : \text{slope} \end{array} \right.$

3 :  $\left\{ \begin{array}{l} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \quad \text{as candidates} \\ 1 : \text{answer with justification} \end{array} \right.$

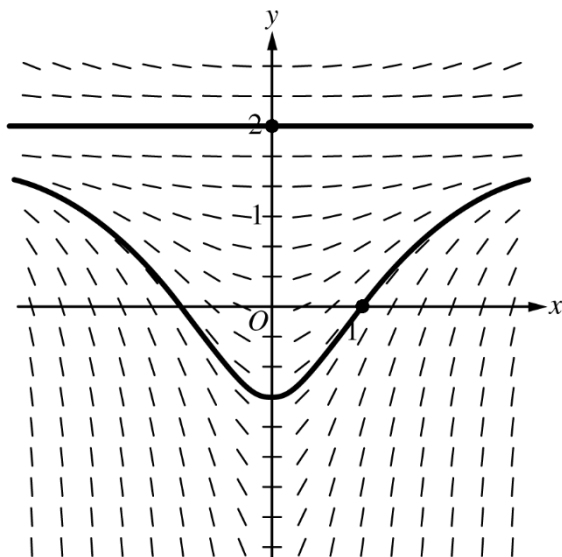
3 :  $\left\{ \begin{array}{l} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \quad \text{and limits equal } 0 \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{array} \right.$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

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**Question 6**

(a)



2 :  $\begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b)  $\left. \frac{dy}{dx} \right|_{(x,y)=(1,0)} = \frac{4}{3}$

An equation for the line tangent to the graph of  $y = f(x)$  at

$x = 1$  is  $y = \frac{4}{3}(x - 1)$ .

$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$

2 :  $\begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$

(c)  $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$

$\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$

$\frac{-1}{y - 2} = \frac{1}{6}x^2 + C$

$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$

$\frac{-1}{y - 2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$

$y = 2 - \frac{6}{x^2 + 2}$

5 :  $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: this solution is valid for  $-\infty < x < \infty$ .