CollegeBoard
Advanced Placement Program

AP®
Calculus BC
1998 Scoring Guidelines

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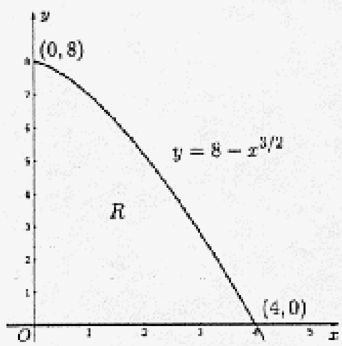
1. Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{\frac{3}{2}}$, the x–axis, and the y–axis.

   (a) Find the area of the region R.

   (b) Find the volume of the solid generated when R is revolved about the x–axis.

   (c) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

(a)



$$A = \int_0^4 \left(8 - x^{3/2}\right)\, dx$$

$$= 8x - \frac{2}{5}x^{5/2}\Big|_0^4 = 32 - \frac{64}{5} = \frac{96}{5} = 19.2$$

$3 \begin{cases} 2\text{: integral} \\ \quad 1\text{: integrand} \\ \quad 1\text{: limits} \\ 1\text{: answer} \end{cases}$

(b) $V = \pi \int_0^4 \left(8 - x^{3/2}\right)^2 dx$

$$= \frac{576\pi}{5} = 115.2\pi \approx 361.911$$

$3 \begin{cases} 2\text{: integral} \\ \quad 1\text{: integrand} \\ \quad 1\text{: limits and constant} \\ 1\text{: answer} \end{cases}$

(c) $\pi \int_0^k \left(8 - x^{3/2}\right)^2 dx = \dfrac{115.2\pi}{2}$

$$\left[\begin{array}{l}\text{or} \\ \pi \int_0^k \left(8 - x^{3/2}\right)^2 dx = \pi \int_k^4 \left(8 - x^{3/2}\right)^2 dx\end{array}\right]$$

$$\int_0^k \left(8 - x^{3/2}\right)^2 dx = 57.6$$

$$\int_0^k \left(64 - 16x^{3/2} + x^3\right) dx = 57.6$$

$$64k - \frac{32}{5}k^{5/2} + \frac{k^4}{4} = 57.6$$

$k \approx 0.995$ or 0.994

$3 \begin{cases} 1\text{: integral with } k \text{ in limits} \\ 1\text{: equates volumes} \\ 1\text{: answer} \end{cases}$

Note: 0/1 for answer in each part if no setup points earned

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2. Let f be the function given by $f(x) = 2xe^{2x}$.

   (a) Find $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to\infty} f(x)$.

   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

   (c) What is the range of f?

   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

(a) $\lim\limits_{x\to-\infty} 2xe^{2x} = 0$

$\lim\limits_{x\to\infty} 2xe^{2x} = \infty$ or DNE

$$2\begin{cases} 1\text{: } 0 \text{ as } x \to -\infty \\ 1\text{: } \infty \text{ or DNE as } x \to \infty \end{cases}$$

(b) $f'(x) = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} = 2e^{2x}(1 + 2x) = 0$

if $x = -1/2$

$f(-1/2) = -1/e$ or -0.368 or -0.367

$-1/e$ is an absolute minimum value because:

(i) $f'(x) < 0$ for all $x < -1/2$ and
$f'(x) > 0$ for all $x > -1/2$

–or–

(ii) $f'(x)$ $\quad - \quad\quad +$

$\qquad\quad -1/2$

and $x = -1/2$ is the only critical number

$$3\begin{cases} 1\text{: solves } f'(x) = 0 \\ 1\text{: evaluates } f \text{ at student's critical point} \\ \quad 0/1 \text{ if not local minimum from} \\ \quad \text{student's derivative} \\ 1\text{: justifies absolute minimum value} \\ \quad 0/1 \text{ for a local argument} \\ \quad 0/1 \text{ without explicit symbolic} \\ \quad \text{derivative} \end{cases}$$

Note: 0/3 if no absolute minimum based on student's derivative

(c) Range of $f = [-1/e, \infty)$
or $[-0.367, \infty)$
or $[-0.368, \infty)$

1: answer

Note: must include the left–hand endpoint; exclude the right–hand "endpoint"

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$

if $x = -1/b$

At $x = -1/b$, $y = -1/e$

y has an absolute minimum value of $-1/e$ for all nonzero b

$$3\begin{cases} 1\text{: sets } y' = be^{bx}(1 + bx) = 0 \\ 1\text{: solves student's } y' = 0 \\ 1\text{: evaluates } y \text{ at a critical number} \\ \quad \text{and gets a value independent of } b \end{cases}$$

Note: 0/3 if only considering specific values of b

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3. Let f be a function that has derivatives of all orders for all real numbers. Assume $f(0) = 5$, $f'(0) = -3$, $f''(0) = 1$, and $f'''(0) = 4$.
   (a) Write the third–degree Taylor polynomial for f about $x = 0$ and use it to approximate $f(0.2)$.
   (b) Write the fourth–degree Taylor polynomial for g, where $g(x) = f(x^2)$, about $x = 0$.
   (c) Write the third–degree Taylor polynomial for h, where $h(x) = \int_0^x f(t)\,dt$, about $x = 0$.
   (d) Let h be defined as in part (c). Given that $f(1) = 3$, either find the exact value of $h(1)$ or explain why it cannot be determined.

(a) $P_3(f)(x) = 5 - 3x + \frac{1}{2}x^2 + \frac{2}{3}x^3$

$f(0.2) \approx P_3(f)(0.2) =$

$$5 - 3(0.2) + \frac{0.04}{2} + \frac{2(0.008)}{3} =$$

4.425

$3 \begin{cases} 2\text{: } 5 - 3x + \frac{1}{2}x^2 + \frac{2}{3}x^3 \\ \quad <-1> \text{ each incorrect term,} \\ \quad \text{extra term, or } + \cdots \\ 1\text{: approximates } f(0.2) \end{cases}$

$<-1>$ for incorrect use of $=$

(b) $P_4(g)(x) = P_2(f)(x^2) = 5 - 3x^2 + \frac{1}{2}x^4$

2: $P_2(f)(x^2)$

$<-1>$ each incorrect or extra term

(c) $P_3(h)(x) = \int_0^x \left(5 - 3t + \frac{1}{2}t^2\right) dt$

$$= \left[5t - \frac{3}{2}t^2 + \frac{1}{6}t^3\right]_0^x$$

$$= 5x - \frac{3}{2}x^2 + \frac{1}{6}x^3$$

$2 \begin{cases} 1\text{: } P_3(h)(x) = \int_0^x P_2(f)(t)\,dt \\ 1\text{: answer} \\ \quad 0/1 \text{ if any incorrect or extra terms} \end{cases}$

(d) $h(1) = \int_0^1 f(t)\,dt$

cannot be determined because $f(t)$ is known only for $t = 0$ and $t = 1$

$2 \begin{cases} 1\text{: } h(1) \text{ cannot be determined} \\ 1\text{: reason} \end{cases}$

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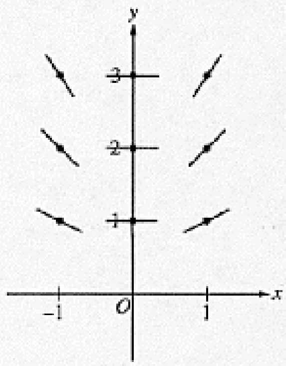
4. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

   (a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.

   (b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

(a)



1: line segments at nine points with negative - zero - positive slope left to right and increasing steepness bottom to top at $x = 1$ and $x = -1$

(b) $f(0.1) \approx f(0) + f'(0)(0.1)$

$$= 3 + \frac{1}{2}(0)(3)(0.1) = 3$$

$f(0.2) \approx f(0.1) + f'(0.1)(0.1)$

$$= 3 + \frac{1}{2}(0.1)(3)(0.1)$$

$$= 3 + \frac{.03}{2} = 3.015$$

2 { 1: Euler's Method equations or equivalent table; 1: answer (not eligible without first point) }

Special Case: 1/2 for first iteration 3.015 and second iteration 3.045

(c) $\frac{dy}{dx} = \frac{xy}{2}$

$$\int \frac{dy}{y} = \int \frac{x}{2}\,dx$$

$$\ln|y| = \frac{1}{4}x^2 + C_1$$

$$y = Ce^{x^2/4}$$

$$3 = Ce^0 \implies C = 3$$

$$y = 3e^{x^2/4}$$

$$f(0.2) = 3e^{.04/4} = 3e^{.01} = 3.030$$

6 {
1: separates variables
1: antiderivative of dy term
1: antiderivative of dx term
1: solves for y
1: solves for constant of integration
1: evaluates $f(0.2)$
}

Note: max 4/6 [1-1-1-0-0-1] if no constant of integration

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5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right), \quad 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

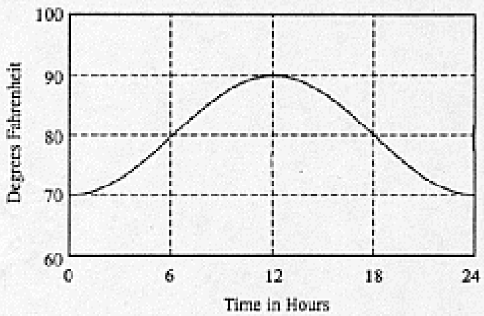
(a) Sketch the graph of F on the grid below.

(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24–hour period?

(a)



1: bell–shaped graph
minimum 70 at $t = 0$, $t = 24$ only
maximum 90 at $t = 12$ only

(b) $$\text{Avg.} = \frac{1}{14-6}\int_6^{14}\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right]dt$$

$$= \frac{1}{8}(697.2957795)$$

$$= 87.162 \text{ or } 87.161$$

$$\approx 87^\circ \text{ F}$$

3 $\begin{cases} \text{2: integral} \\ \quad \text{1: limits and } 1/(14-6) \\ \quad \text{1: integrand} \\ \text{1: answer} \\ \quad 0/1 \text{ if integral not of the form} \\ \quad \dfrac{1}{b-a}\displaystyle\int_a^b F(t)\,dt \end{cases}$

(c) $$\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right] - 78 \ge 0$$

$$2 - 10\cos\left(\frac{\pi t}{12}\right) \ge 0$$

$$\left.\begin{matrix} 5.230 \\ \text{or} \\ 5.231 \end{matrix}\right\} \le t \le \left\{\begin{matrix} 18.769 \\ \text{or} \\ 18.770 \end{matrix}\right.$$

2 $\begin{cases} \text{1: inequality or equation} \\ \text{1: solutions with interval} \end{cases}$

(d) $$C = 0.05\int_{\substack{5.231 \\ \text{or} \\ 5.230}}^{\substack{18.770 \\ \text{or} \\ 18.769}}\left(\left[80 - 10\cos\left(\frac{\pi t}{12}\right)\right] - 78\right)dt$$

$$= 0.05(101.92741) = 5.096 \approx \$5.10$$

3 $\begin{cases} \text{2: integral} \\ \quad \text{1: limits and } 0.05 \\ \quad \text{1: integrand} \\ \text{1: answer} \\ \quad 0/1 \text{ if integral not of the form} \\ \quad k\displaystyle\int_a^b (F(t) - 78)\,dt \end{cases}$

6. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x–coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with initial condition $x(0) = -4$.

(a) Find $x(t)$ in terms of t.

(b) Find $\frac{dy}{dt}$ in terms of t.

(c) Find the location and speed of the particle at time $t = 4$.

(a) $x(t) = \int \frac{1}{\sqrt{2t+1}}\, dt$

$x(t) = \sqrt{2t+1} + C$

$x(0) = -4 = 1 + C \implies C = -5$

$x(t) = \sqrt{2t+1} - 5$

$$3 \begin{cases} 1\text{: } x(t) = \int \frac{dt}{\sqrt{2t+1}} \\ 1\text{: } x(t) = \sqrt{2t+1} + C \\ 1\text{: evaluates } C \end{cases}$$

(b) $y = x^3 - 3x$

$\frac{dy}{dt} = 3x^2 \frac{dx}{dt} - 3\frac{dx}{dt}$

$= (3x^2 - 3)\frac{dx}{dt}$

$= \left[3\left(\sqrt{2t+1} - 5\right)^2 - 3\right]\left[\frac{1}{\sqrt{2t+1}}\right]$

2: answer

<−1> each error

Note: failure to express $\frac{dy}{dt}$ solely in terms of t is a single error

(c) $x(4) = \sqrt{9} - 5 = -2$

$y(4) = (-2)^3 - 3(-2) = -2$

Location at $t = 4$ is $(-2, -2)$

$\left.\frac{dx}{dt}\right|_{t=4} = \frac{1}{3}$

$\left.\frac{dy}{dt}\right|_{t=4} = \frac{3(3-5)^2 - 3}{3} = 3$

$\text{Speed} = \sqrt{\left(\frac{1}{3}\right)^2 + 3^2} = \sqrt{\frac{82}{9}} = 3.018$

$$4 \begin{cases} 1\text{: position} \\ 1\text{: evaluates } \frac{dx}{dt} \text{ and } \frac{dy}{dt} \text{ at } t = 4 \\ 1\text{: uses speed formula} \\ 1\text{: answer} \end{cases}$$