AP Calculus BC
1999 Sample Student Responses

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A₁

4. The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

(a) Write the third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

$$T_3(x) = -3 + 5(x-2) + \frac{3}{2}(x-2)^2 + -\frac{4}{3}(x-2)^3$$

$$f(1.5) \approx -3 + 5(1.5-2) + \frac{3}{2}(1.5-2)^2 - \frac{4}{3}(1.5-2)^3$$

$$= -4.958$$

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 3$ for all x in the closed interval $[1.5, 2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \neq -5$.

$$R_3(1.5) = \frac{f^{(4)}(z)}{4!}(1.5-2)^4 \quad \text{for some } z,\ 1.5 \le z \le 2.$$

Thus, $R_3(1.5) \le \frac{3}{4!}(1.5-2)^4 = .0078125$

Thus, $-4.958 - .0078125 \le f(1.5) \le -4.958 + .0078125$

$$-4.966 \le f(1.5) \le -4.950$$

Thus, $f(1.5) \neq -5$

Continue problem 4 on page 11.

(c) Write the fourth-degree Taylor polynomial, $P(x)$, for $g(x) = f(x^2 + 2)$ about $x = 0$. Use P to explain why g must have a relative minimum at $x = 0$.

$P(x) = -3 + 5x^2 + \frac{3}{2}x^4$

Since the coefficent of x is 0, $\frac{g'(0)}{1!} = 0$, so $g'(0) = 0$.
Since the coefficent of x^2 is 5, $\frac{g''(0)}{2!} = 5$, so $g''(0) = 10$

Thus, since $g''(0)$ is positive and $g'(0) = 0$, $P(x)$ must have a relative minimum at $x = 0$ by the second derivative test.

D_1

4. The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

(a) Write the third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

$$f(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3$$

$$= -3 + 5(x-2) + \frac{3}{2!}(x-2)^2 - \frac{8}{3!}(x-2)^3$$

$$f(1.5) = -4.958$$

(b) The fourth derivative of f satisfies the inequality $|f^{(4)}(x)| \le 3$ for all x in the closed interval $[1.5, 2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \ne -5$.

$|error| \le a_{n+1}$

$|error| \le a_4$

$|error| \le \frac{3}{4!}(x-2)^4$

$|error| \le \frac{3}{24}(1.5-2)^4$

$|error| \le .0078$

The truncation error is no greater than .0078

$\therefore \quad -4.958 - .0078 \le f(1.5) \le -4.958 + .0078$

$-4.9658 \le f(1.5) \le -4.9502$

$\therefore \quad f(1.5) \ne -5$

Continue problem 4 on page 11.

(c) Write the fourth-degree Taylor polynomial, $P(x)$, for $g(x) = f\left(x^2 + 2\right)$ about $x = 0$. Use P to explain why g must have a relative minimum at $x = 0$.

$$P(x) = -3 + 5(x^2+2-2) + \frac{3}{2!}(x^2+2-2)^2$$
$$= -3 + 5x^2 + \frac{3}{2!}x^4$$
$$P'(x) = 10x + 6x^3$$
$$0 = 2x(5 + 3x^2)$$
$$x = 0$$

$-$ | $+$ $P'(x) = g'(x)$

0

$\therefore$ $g(x)$ has a relative minimum at $x = 0$

E1

4. The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

(a) Write the third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

$$\boxed{f(x) = -3 + 5(x-2) + \frac{3(x-2)^2}{2!} - \frac{8(x-2)^3}{3!}}$$

$$f(1.5) = -3 + 5(1.5-2) + \frac{3(1.5-2)^2}{2!} - \frac{8(x-2)^3}{3!}$$

$$\boxed{f(1.5) = -4.958}$$

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 3$ for all x in the closed interval $[1.5, 2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \neq -5$.

$$f^3(x) > f^4(x)$$

$$\frac{-8(-.5)^3}{3!} > \frac{f^4(-.5)^4}{4!}$$

$$1.667 > .002604(f^4)$$

since there is an error of at least .002604, $f(1.5) \neq -5$.

Continue problem 4 on page 11.

(c) Write the fourth-degree Taylor polynomial, $P(x)$, for $g(x) = f\left(x^2 + 2\right)$ about $x = 0$. Use P to explain why g must have a relative minimum at $x = 0$.

$g(x) = f(x^2+2)$

$P(x) = -3 + \dfrac{5(x^2+2-2)}{} + \dfrac{3(x^2+2-2)^2}{2!}$

$\boxed{P(x) = -3 + 5x^2 + \dfrac{3x^4}{2!}}$

$P'(x) = 10x + \dfrac{12x^3}{2}$

$P'(x) = 10x + 6x^3$

$0 = 2x(5+3x^2)$

$x = 0$ other 2 roots are complex

$P'(x)$ — 0 +

Because $P'(x)$ has a relative min at $x=0$, so does $g(x)$

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