AP Calculus BC
1999 Sample Student Responses

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A,

6. Let f be the function whose graph goes through the point (3, 6) and whose derivative is given by $f'(x) = \frac{1 + e^x}{x^2}$.

(a) Write an equation of the line tangent to the graph of f at $x = 3$ and use it to approximate $f(3.1)$.

slope @ $x=3$: $\frac{1+e^3}{9}$

$$y - 6 = \left(\frac{1+e^3}{9}\right)(x-3)$$

$$y - 6 = \left(\frac{1+e^3}{9}\right)(3.1-3)$$

$$f(3.1) \approx 6 + \left(\frac{1+e^3}{9}\right)(.1)$$

$$f(3.1) \approx 6.234$$

(b) Use Euler's method, starting at $x = 3$ with a step size of 0.05, to approximate $f(3.1)$. Use f'' to explain why this approximation is less than $f(3.1)$.

x	y	slope	Δy
3	6	$\frac{1+e^3}{9}$	.117
3.05	6.117	2.377	
3.1	6.236		

$f(3.1) \approx 6.236$

$$f''(x) = \frac{x^2(e^x) - (1+e^x)2x}{x^4}$$

$f''(x)$: $-$ at 3, $+$ CCU

since $f''(x)$ is positive when $x > 3$ the graph of f is concave up, thus the tangent lines are below the actual graph of f and the values found by using the tangent lines are lower than the actual values

Continue problem 6 on page 15.

(c) Use $\int_3^{3.1} f'(x)dx$ to evaluate $f(3.1)$.

$\int_3^{3.1} f'(x)$

$= f(3.1) - f(3)$ $\qquad f(3)=6$

$\int_3^{3.1} \frac{1+e^x}{x^2}$

$= .238$

$.238 = f(3.1) - 6$

$\boxed{f(3.1) = 6.238}$

END OF EXAMINATION

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D1

6. Let f be the function whose graph goes through the point (3, 6) and whose derivative is given by $f'(x) = \frac{1 + e^x}{x^2}$.

(a) Write an equation of the line tangent to the graph of f at $x = 3$ and use it to approximate $f(3.1)$.

$f'(3) = \frac{1+e^3}{9} \approx 2.343$

tangeant line → $y - 6 = 2.343(x-3)$ //

sub in $x = 3.1$

$y - 6 = 2.343(3.1-3)$

$y \approx f(3.1) \approx 6.234$ //

(b) Use Euler's method, starting at $x = 3$ with a step size of 0.05, to approximate $f(3.1)$. Use f'' to explain why this approximation is less than $f(3.1)$.

at (3, 6) slope = $\frac{1+e^3}{9} = 2.343$

↓

$(3.05, 6 + 0.05(2.343))$

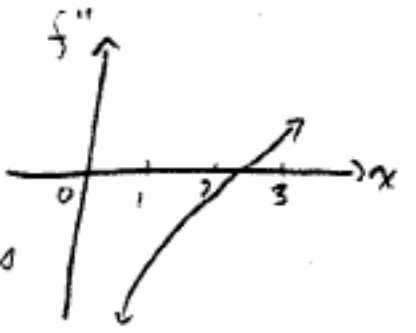
$= (3.05, 6.117)$ slope = $\frac{1+e^{3.05}}{3.05^2} = 2.377$

↓

$(3.1, 6.117 + 0.05(2.377))$

$= (3.1, 6.236) \rightarrow f(3.1) \approx 6.236$ //

$f''(x) = \frac{x^2e^x - 2x(1+e^x)}{x^4}$ → graph on calculator



The graph of $f''(x)$ is positive for $3 \le x \le 3.1$, which means f is concave up ($\cup$) ∴ Any tangeant line to f would lie under the graph, making the approximation less than the actual value //

Continue problem 6 on page 15.

D_2

(c) Use $\int_3^{3.1} f'(x)dx$ to evaluate $f(3.1)$.

$$\int_3^{3.1} \frac{1+e^x}{x^2}\,dx$$

evaluate in calculator ...

$$\approx 0.2378$$

$$f(3.1) \approx \frac{1}{3.1-3}\int_3^{3.1} f'(x)\,dx$$

$$\approx \frac{1}{0.1}(0.2378)$$

$$= 2.378 //$$

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E1

6. Let f be the function whose graph goes through the point (3, 6) and whose derivative is given by $f'(x) = \frac{1 + e^x}{x^2}$.

(a) Write an equation of the line tangent to the graph of f at $x = 3$ and use it to approximate $f(3.1)$.

$f'(x) = \frac{1+e^x}{x^2}$ $\quad f'(3) = \frac{1+e^3}{9} = m_{tangent}$

$\boxed{y - 6 = \frac{1+e^3}{9}(x-3)}$ = tangent line

$y - 6 = \frac{1+e^3}{9}(3.1-3)$

$y - 6 = \frac{1+e^3}{9}(0.1)$

$\boxed{y \approx 6.234 \text{ at } x=3.1 \text{ by approximation}}$

(b) Use Euler's method, starting at $x = 3$ with a step size of 0.05, to approximate $f(3.1)$. Use f'' to explain why this approximation is less than $f(3.1)$.

$x_0 = 3 \quad y_0 = 6$

$x_1 = 3.05 \quad y_1 = 6 + 0.05\left(\frac{1+e^3}{3^2}\right)$

$y_1 = 6.11714187 18$

$x_2 = 3.1 \quad y_2 = 6.11714187 18 + 0.05\left(\frac{1+e^{3.05}}{(3.05)^2}\right)$

$\boxed{f(3.1) \approx y_2 = 6.236}$

This approximation is less than $f(3.1)$ because we are underapproximating it.

Continue problem 6 on page 15.

E_2

(c) Use $\int_3^{3.1} f'(x)dx$ to evaluate $f(3.1)$.

$$\int_3^{3.1} \frac{1+e^x}{x^2}\,dx$$

$$\int_3^{3.1} \frac{1}{x^2}(1+e^x)^{-}dx$$

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