AP Calculus BC
2000 Student Samples

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A1

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$\vec{v} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \left\langle 1 - \frac{1}{t^2}, 2 + \frac{1}{t^2} \right\rangle$$

$$\vec{a} = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle = \left\langle 2t^{-3}, -2t^{-3} \right\rangle$$

$$\Rightarrow \vec{a}(3) = \left\langle \frac{2}{27}, -\frac{2}{27} \right\rangle$$

Work for problem 4(b)

$$x(t) = \int \left(\frac{dx}{dt}\right) dt = \int (1 - t^{-2})\, dt = t + t^{-1} + C$$

$$x(1) = 2 + C = 2 \Rightarrow C = 0$$

$$y(t) = \int \left(\frac{dy}{dt}\right) dt = \int (2 + t^{-2})\, dt = 2t - t^{-1} + C$$

$$y(1) = 1 + C = 6 \Rightarrow C = 5$$

$$\Rightarrow (x(t), y(t)) = (t + t^{-1}, 2t - t^{-1} + 5)$$

$$\therefore (x(3), y(3)) = \left(\frac{10}{3}, \frac{32}{3}\right)$$

Continue problem 4 on page 11.

Work for problem 4(c)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2 + t^{-2}}{1 - t^{-2}} = \frac{2t^2 + 1}{t^2 - 1} = 8$$

$$\Leftrightarrow \quad 6t^2 = 9$$

$$\Leftrightarrow \quad t = \sqrt{\frac{3}{2}}, \quad t > 0$$

Work for problem 4(d)

$$\lim_{t \to \infty} \left(\frac{dy}{dx}\right) = \lim_{t \to \infty} \left(\frac{2t^2 + 1}{t^2 - 1}\right)$$ both numerator & denominator are undefined, so by L'Hopital's Rule,

$$= \lim_{t \to \infty} \left(\frac{4t}{2t}\right) = \lim_{t \to \infty} 2 = 2$$

∴ the slope of the line ~~that~~ y(x) approaches is 2

C1

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

velocity $= (1 - t^{-2}, 2 + t^{-2})$

acc $= (2t^{-3}, -2t^{-3})$

$\left(\frac{2}{t^3}, -\frac{2}{t^3}\right)$ at $t = 3$

$\boxed{\left(\frac{2}{27}, \frac{-2}{27}\right)}$

Work for problem 4(b)

velocity $= (1 - t^{-2}, 2 + t^{-2})$

$t = 1$
pos $(2, 6)$

$x(t)$

$\int (1 - t^{-2})\,dt$

$t + t^{-1} + C$

$2 = 1 + 1 + C$

$C = 0$

$x(t) = t + \frac{1}{t}$

$y(t)$

$\int (2 + t^{-2})\,dt$

$2t - t^{-1} + C$

$6 = 2 - 1 + C$

$6 = 1 + C$

$C = 5$

$y(t) = 2t + \frac{1}{t} + 5$

$6 + \frac{1}{3} + 5 =$

at $t = 3$

position $\boxed{\left(\frac{4}{3}, 11\frac{1}{3}\right)}$

Continue problem 4 on page 11.

Work for problem 4(c)

$$\frac{dx}{dt} = 1 - t^{-2} \qquad \frac{dy}{dt} \quad 2 + t^{-2}$$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} = \frac{2 + t^{-2}}{1 - t^{-2}} \qquad \frac{2 + t^{-2}}{1 - t^{-2}} = 8$$

$$2 + t^{-2} = 8 - 8t^{-2}$$

$$9t^{-2} = 6$$

$$\frac{9}{t^2} = 6 \qquad 6t^2 = 9$$

$$t^2 = \frac{9}{6}$$

$$t = \sqrt{\frac{9}{6}} = \boxed{\frac{3}{\sqrt{6}}}$$

Work for problem 4(d)

$$\lim_{t \to \infty} \frac{2 + t^{-2}}{1 - t^{-2}} = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$$

take deriv of top + bottom L'Hopital's rule

$$\lim_{t \to \infty} \frac{-2t^{-3}}{2t^{-3}} = \boxed{-1}$$

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CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

$-t^{-2}$
$+2t^{-3}$

Work for problem 4(a)

$v(t) = \left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$ $\Rightarrow v(t) = (x, y)$

$a(t) = (dx, dy)$

$a(t) = \left(\frac{1}{2t^3}, -\frac{1}{2t^3}\right)$

$a(3) = \left(\frac{1}{2(3)^3}, \frac{-1}{2(3)^3}\right)$

$a(3) = \left(\frac{1}{54}, \frac{-1}{54}\right)$

Work for problem 4(b)

$t^{-1} \Rightarrow -t^{-2}$

$v(t) = \left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right) \Rightarrow v(t) = (dx, dy)$

$p(t) = (x, y)$

$p(t) = \left(t + \frac{1}{t}, 2t - \frac{1}{t}\right)$

$p(3) = (3\frac{1}{3}, 5\frac{2}{3})$

Continue problem 4 on page 11.

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Work for problem 4(c)

$$\frac{dy}{dx} = 8 = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$$

$$8 - \frac{8}{t^2} = 2 + \frac{1}{t^2}$$

$$6 = \frac{9}{t^2}$$

$$6t^2 = 9$$

$$t^2 = 3/2$$

$$t = \sqrt{3}/\sqrt{2}$$

Work for problem 4(d)

$$\lim_{t \to \infty} \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$$