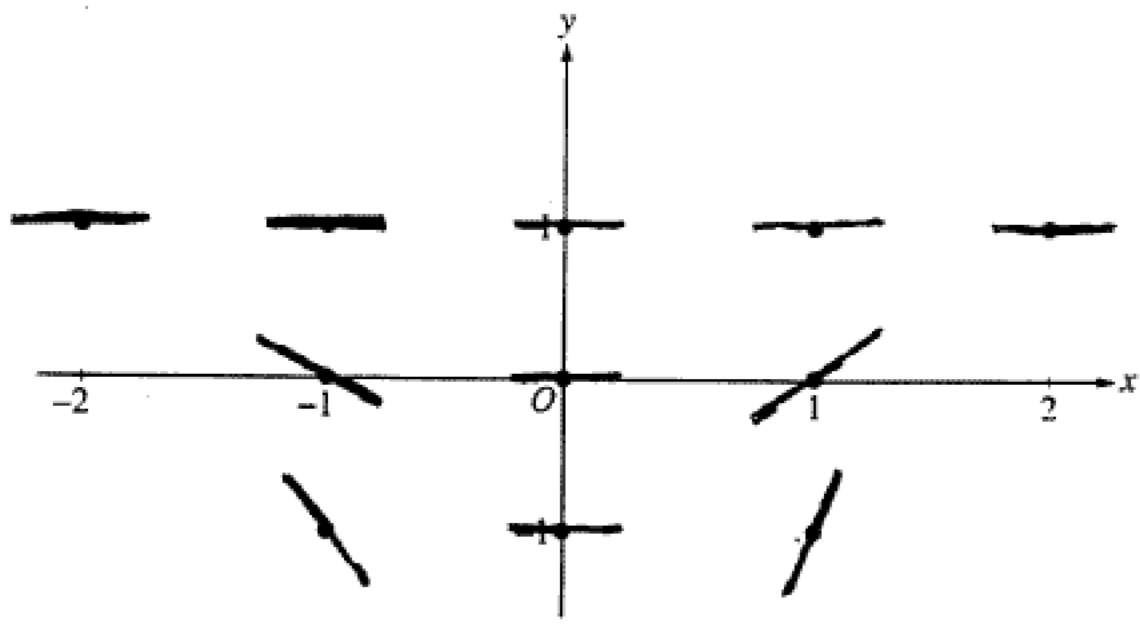
AP Calculus BC
2000 Student Samples

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

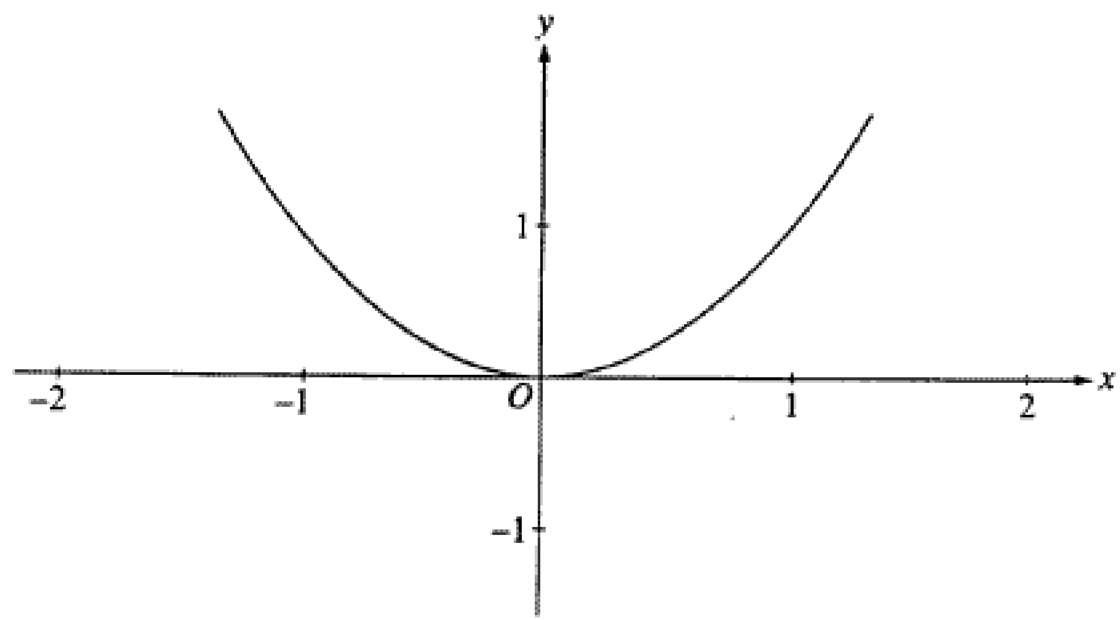
The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT™, the Advanced Placement Program® (AP®), and Pacesetter®. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

Work for problem 6(a)

at (0,0) = 0
(0,1) = 0
(0,-1) = 0
(1,0) = 1
(1,1) = 0
(1,-1) = 4
(2,1) = 0
(-1,1) = 0
(-2,1) = 0
(-1,0) = -1
(-1,-1) = -4



Work for problem 6(b)



this graph passes through the points (-1, 1) and (1,1) and has a slope whose absolute value is greater than 0. However, in the slope field, the slope at these points is 0. therefore, this graph cannot match the differential equation

Continue problem 6 on page 15.

A₂

Work for problem 6(c)

$\frac{dy}{dx} = x(y-1)^2$

$\frac{dy}{(y-1)^2} = x\,dx$

integrate: $\frac{-1}{(y-1)} = \frac{1}{2}x^2 + C$

$\frac{-1}{(-1-1)} = \frac{1}{2}(0)^2 + C$

$\frac{1}{2} = 0 + c$

$c = \frac{1}{2}$

$\frac{-1}{y-1} = \frac{1}{2}x^2 + \frac{1}{2}$

$\frac{-1}{\frac{1}{2}(x^2+1)} = y-1$

$\boxed{y = \frac{-2}{x^2+1} + 1}$

Work for problem 6(d)

when $x = 0$, $y = -1$

as x approaches ∞ or $-\infty$, y goes to 1

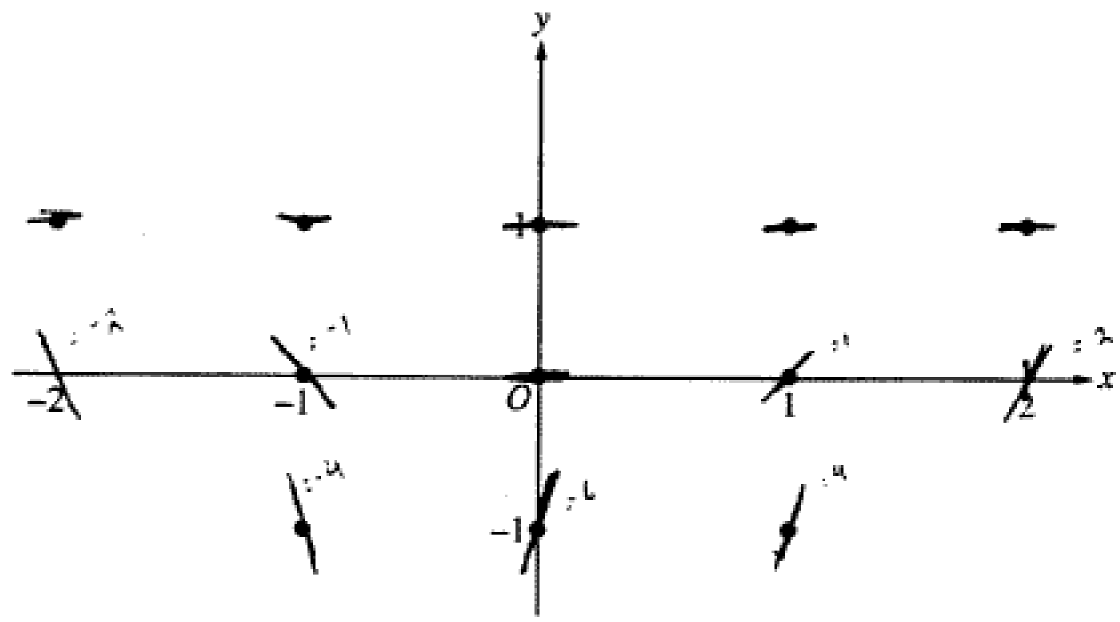
$\boxed{[-1, 1)}$

END OF EXAMINATION

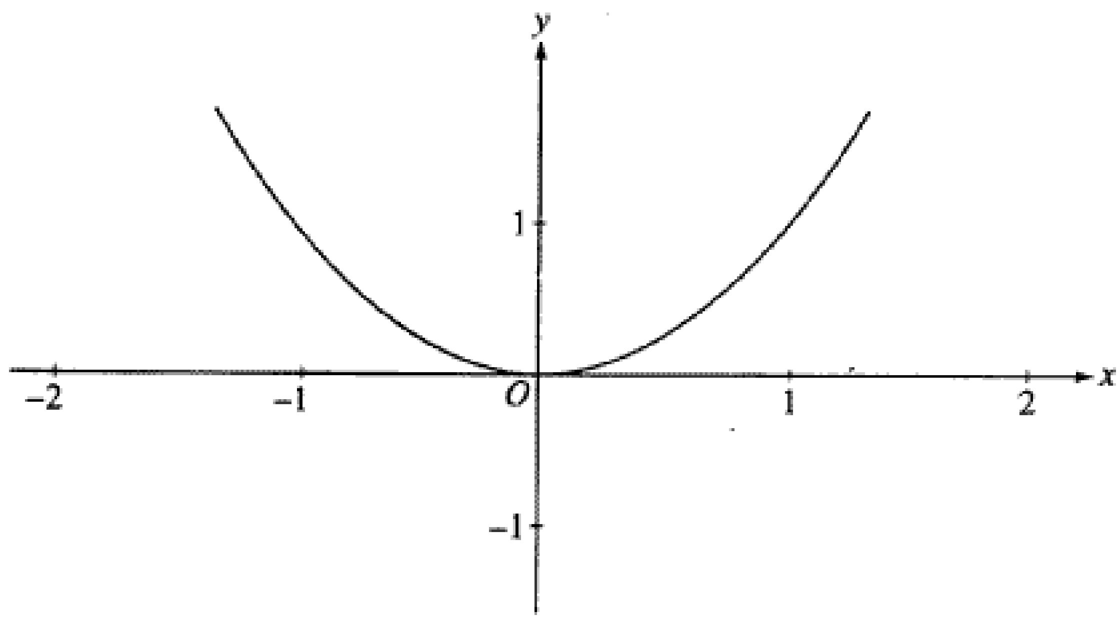
THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON <u>ALL</u> AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.**

Work for problem 6(a)



Work for problem 6(b)



This graph can not be a solution because at the points (-1, 1) and (1, 1) the slope of the function must be zero. In this particular graph there is a slope not equalling 0 at these two points.

Continue problem 6 on page 15.

6 6 6 6 6 6 6 6 6 6

D_2

Work for problem 6(c)

$$\frac{dy}{(y-1)^2} = x\,dx$$

$u = y-1$ $\quad \int \frac{du}{u^2} = \int x\,dx$

$\int u^{-2}$

$$-u^{-1} = \frac{1}{2}x^2$$

$$\frac{-1}{(y-1)} = \frac{1}{2}x^2 + c$$

$$-\frac{1}{(-1-1)} = \frac{1}{2}(0)^2 + c \qquad -\frac{1}{-2} = c \qquad c = \frac{1}{2}$$

$$-\frac{1}{(y-1)} = \frac{1}{2}x^2 + \frac{1}{2}$$

$$-\frac{1}{y-1} = \frac{1}{2}(x^2+1)$$

$$-(y-1) = \frac{2}{x^2+1}$$

$$y = -\frac{2}{(x^2+1)} + 1$$

Work for problem 6(d)

range $: -1 < y < 1$

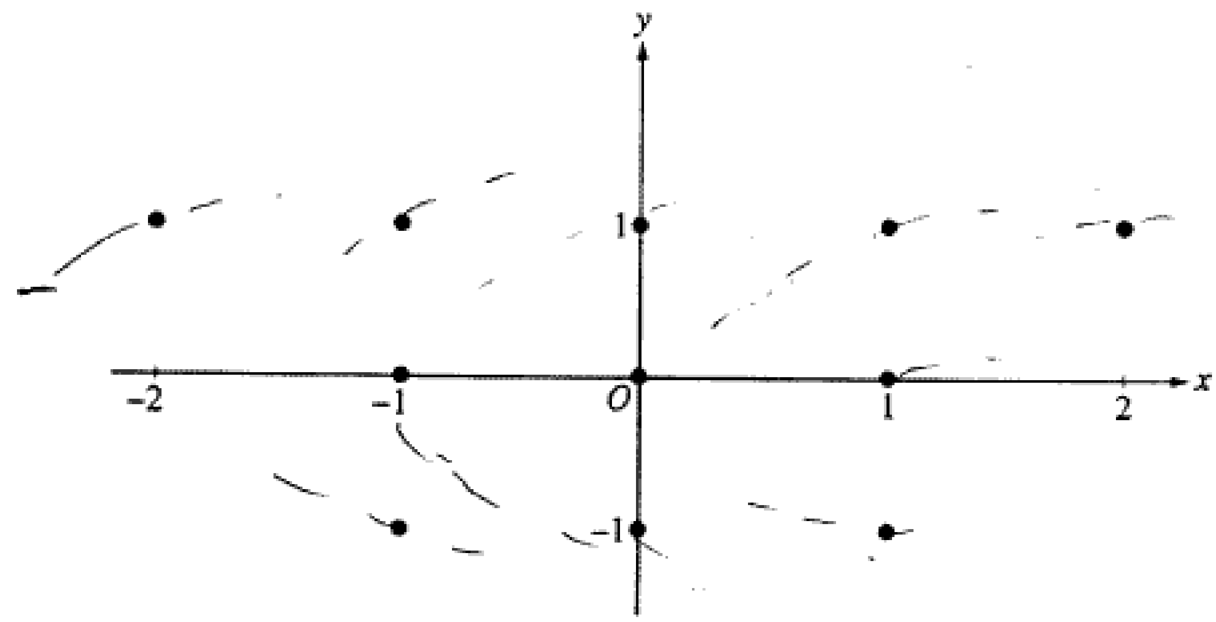
END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

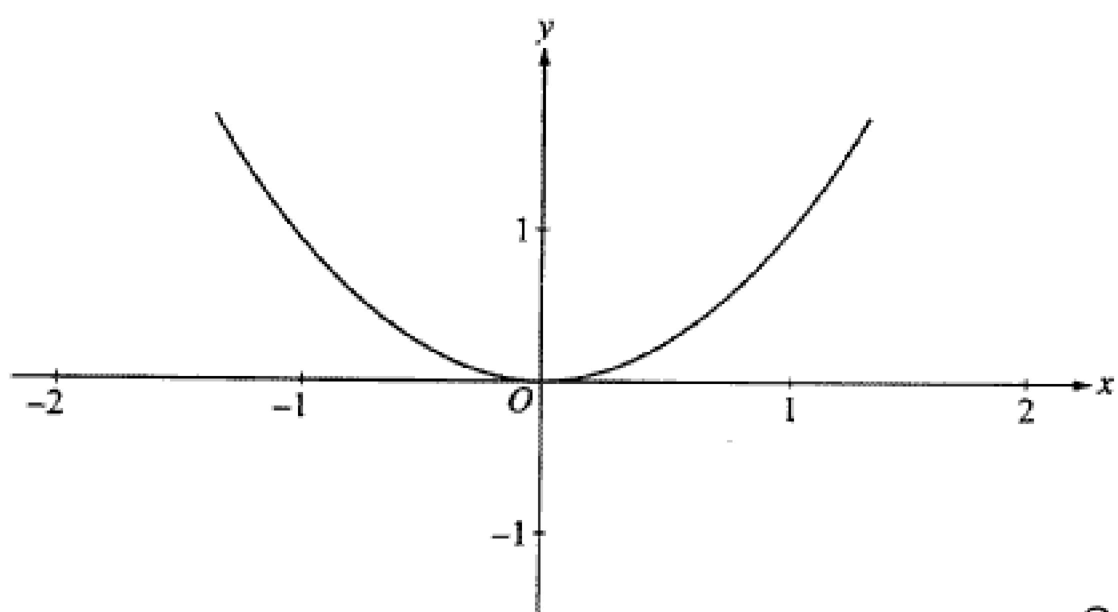
- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.**

$\frac{dy}{dx} = x(y-1)^2 \rightarrow y^2x - 2xy + x$

Work for problem 6(a)



Work for problem 6(b)



Parabolas such as the one shown above have 2 values of x for each unique value y. However, the differential equation from above shows that there are more values for y than x, so the shape ~~so be~~ should ~~be~~ lie sideways.

Continue problem 6 on page 15.

F_2

Work for problem 6(c)

$f(0) = -1$

$\int \frac{dy}{(y-1)^2} = \int x\,dx \to \int \frac{dy}{y^2-2y+1} = \int x\,dx$

$\frac{\cancel{(x-1)}(x+1)}{\cancel{x}}$

$\frac{1}{2}x^2 = -(y-1)^{-1} + C$

$\frac{1}{2}x^2 + C = -\frac{1}{y-1} \to$ $(0,-1)$: $0 + C = -\frac{1}{-1-1} \to C = -\frac{1}{(-2)} = \frac{1}{2}$

$\frac{1}{2}x^2 + \frac{1}{2} = -\frac{1}{y-1} \to -\frac{1}{2}x^2 - \frac{1}{2} = \frac{1}{y-1} \to \frac{-x^2-1}{2} = \frac{1}{y-1}$

$\frac{2}{-(x^2+1)} = (y-1) \to \frac{-2}{x^2+1} + 1 = y$ $\quad \frac{-2+x^2+1}{x^2+1} \Rightarrow \boxed{\frac{x^2-1}{x^2+1} = y}$

Work for problem 6(d)

range:

* every value of y allowed because x^2+1 can never equal 0, thus y will never be ∞

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON <u>ALL</u> AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.**