AP® Calculus BC
2001 Scoring Commentary

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Question 1

This problem involved parametric motion in the plane. It presented the student with the velocity components of an object moving along a curve in the plane and the object's position at a given time.

Part (a) asked for an equation of the line tangent to the curve, which required the student to determine the slope from the velocity components. Part (b) asked for a calculation of the object's speed and part (c) asked for the total distance traveled by the object over a given time interval. Part (d) required the student to integrate each of the velocity components and use the position of the object at $t = 2$ to calculate the coordinates of the object's position at a specific time. The velocity components of the object were functions that required the use of the numerical integration capabilities of a calculator in parts (c) and (d).

The mean score was 2.69.

<u>Sample B</u>

The student earned all 9 points. The student communicated a clear understanding of the Fundamental Theorem in part (d).

<u>Sample D</u>

The student earned 7 points: 1 point in part (a), 1 point in part (b), 2 points in part (c), and 3 points in part (d). The student failed to calculate a value in part (c). The student made a calculation error in finding the value for $y(3)$ in part (d).

Question 2

This problem presented the student with tabular data representing water temperature readings in a pond recorded at regular time intervals.

Part (a) asked for an approximation of a derivative (with appropriate units) and part (b) asked for a (trapezoidal) approximation of a definite integral representing the average value of the temperature using this tabular data. In part (c), a function model for the water temperature was introduced. The student was asked to calculate a derivative analytically and to provide an interpretation of its meaning in this physical context. Part (d) required the student to calculate the average value of the function model over the time interval in part (b). This problem reflected the increased emphasis on working with multiple representations of functions. It illustrated two ways of working with data: approximations based on actual data values and analytic work based on a model approximating the data.

The mean score was 5.27.

<u>Sample A</u>

The student earned all 9 points.

<u>Sample C</u>

The student earned 7 points: 1 point in part (a), 2 points in part (b), 1 point in part (c), and 3 points in part (d). The second point in part (a) was not earned because units were omitted. The student did not earn the second point in part (c). The second point in part (c) required the student to use a word or phrase to indicate that the temperature is decreasing, and this student did not.

Question 3

This problem presented the student with a car's initial velocity and a piecewise linear graphical representation of the car's acceleration over a time interval. The different parts of the problem asked for interpretations and conclusions regarding the car's velocity. This required the student to both recognize the acceleration graph as that of the derivative of the velocity and to reason using this graphical representation of the derivative. It is possible that a student might have chosen to obtain a piecewise formula for the acceleration function in order to work analytically, but it would have been more efficient to reason directly from the graph.

Part (a) asked for an interpretation of the rate of change of the velocity at a given time (obtained by the sign of the value of the acceleration function). Parts (b), (c), and (d) each required calculating accumulated changes in velocity in terms of definite integrals of the acceleration function, all of which could be computed directly by summing the signed areas of triangles, rectangles and/or trapezoids. Part (c) also involved an analysis of local extrema and endpoint analysis in finding the time at which an absolute maximum velocity occurs. All parts of the problem asked for supporting reasons and justifications for the students' conclusions.

The mean score was 4.17.

Sample A

The student earned all 9 points.

Sample D

The student earned 7 points: 1 point in part (a), 2 points in part (b), 3 points in part (c), and 1 point in part (d). The fourth point in part (c) was not earned because the student did not properly eliminate $t = 18$ from consideration. The second point in part (d) was not earned because the student did not justify the answer provided.

Question 4

This problem presented the student with a derivative of a function *h* and an initial value of the function. Given this information, a student might have attempted to determine the function *h* explicitly by first antidifferentiating h' and then using the given initial condition to determine the constant of integration. However, the function *h* had a discontinuity at $x = 0$ and so the initial condition only determined the function for $x > 0$. For all parts of this problem the student could have used information supplied by the given derivative h' or its derivative h''.

Part (a) asked for the critical values and for an extrema analysis with justification. This justification could have involved either the first or second derivative tests. Part (b) asked for a concavity analysis that could be in terms of the second derivative or in terms of the increasing/decreasing behavior of the first derivative. Part (c) required the student to find the equation of the line tangent to the graph of *h* at the point defined by the initial condition. Part (d) tied parts (b) and (c) together by asking for a geometric analysis of the relationship of the line tangent to the graph of *h* found in part (c) by using the concavity information from part (b).

The mean score was 4.70.

Sample A

The student earned all 9 points.

Sample D

The student earned 7 points: 4 points in part (a), 2 points in part (b), and 1 point in part (c). The student correctly used the second derivative test in part (a) but failed to exclude “0” in part (b). The point in part (d) was not earned because the student did not make it clear that the curve was concave up on the given interval.

Question 5

This problem presented the student with a differential equation relating a function f to its derivative f'. The student was also presented with both an initial condition and a limiting condition (in other words, the asymptotic behavior of $f(x)$ as x approaches infinity).

Part (a) asked for an improper integral of the expression for f' given by the differential equation. The solution of this problem required the student to recognize the integrand as being f' and use the Fundamental Theorem of Calculus to employ both the initial and limiting conditions in the calculation. Part (b) asked the student to use Euler's method to approximate the value of the function at another point. Part (c) asked the student to explicitly solve the given separable differential equation with initial condition. It is possible that a student might have used this explicit solution to approach parts (a) and/or (b).

The mean score was 5.61.

<u>Sample A</u>

The student earned all 9 points.

<u>Sample D</u>

The student earned 7 points: 2 points in part (b) and 5 points in part (c). The student earned no points in part (a). The student had an incorrect antiderivative, and the polynomial was not eligible for the answer point because it did not involve the given limit information.

Question 6

This problem presented the student with a power series representation for a function.

Part (a) asked for the interval of convergence of the given power series. Part (b) asked for a limit of an indeterminate form $\left(\frac{0}{0}\right)$ involving the function defined by the power series. A student might have used L'Hôpital's Rule or simplified the indeterminate form. In either case, some formal manipulation of the power series was needed, either by differentiation or by algebraic means. Part (c) also required the student to formally manipulate the given power series in a way that could have been used to calculate the value of a definite integral. This resulted in a geometric series that the student was asked to evaluate in part (d).

The mean score was 3.68.

<u>Sample B</u>

The student earned all 9 points. In part (a), a test for divergence at the endpoints was not required.

<u>Sample D</u>

The student earned 7 points: 3 points in part (a), 1 point in part (b), 2 points in part (c), and 1 point in part (d). The last point in part (a) was not earned because the student came to an incorrect conclusion about convergence at $x = -3$. The last point in part (c) was not earned because the student did not give a general term for the infinite series.