AP® Calculus BC
2002 Sample Student Responses
Form B

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A1

Work for problem 2(a)

a) $P'(9) = 1 - 3(e)^{-0.2\sqrt{9}}$

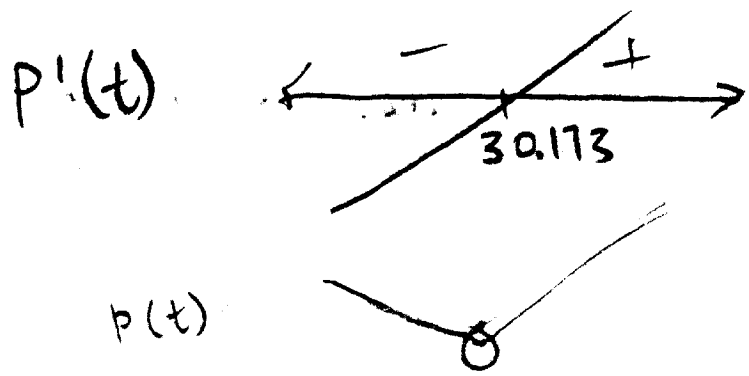
$= -0.646$ gallons/day.

No. $P'(9)$ is negative, so the amount of pollutant is decreasing.

Work for problem 2(b)

b) $p'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$

$t = 30.173$



minimum at $t = 30.173$

A_2

Work for problem 2(c)

$$50 + \int_0^{30.173} P'(t)\,dt$$

$$= 50 - 14.895$$

$$= 35.104 \text{ gallons}$$

At day 30, there will be 35.104 gallons of pollutant left, and $35.104 < 40$, ∴ it will be safe.

Work for problem 2(d)

$$P'(0) = 1 - 3e^{-0.2(0)}$$

$$= 1 - 3e^{0}$$

$$= 1 - 3$$

$$= -2$$

$P(0) = 50$

$y - 50 = -2(x)$

$y = -2x + 50$

$y \leq 40$

$-2x + 50 \leq 40$

$-2x \leq -10$

$x \geq 5$

It predicts that at $t = 5$ the lake will become safe.

GO ON TO THE NEXT PAGE.

C1

Work for problem 2(a)

$P'(9) = 1 - 3e^{-0.2\sqrt{9}} = -0.646$ gallons/day

the level of pollutant is decreasing because the rate is negative, as it is decreasing.

Work for problem 2(b)

gallons of pollutant at a min when $P'(t) = 0$

$1 - 3e^{-0.2\sqrt{t}} = 0$

$3e^{-0.2\sqrt{t}} = 1$

$e^{-0.2\sqrt{t}} = 1/3$

$-0.2\sqrt{t} = \ln 1/3$

$\sqrt{t} = \frac{\ln 1/3}{-0.2}$

$t = \left(\frac{\ln 1/3}{-0.2}\right)^2$

$= 30.174$

≈ 30 days

C_2

Work for problem 2(c)

no. of gallons present at the lake

$$= 50 + \int_0^{30.174} (1 - 3e^{-0.2\sqrt{t}})\,dt$$

$$= 50.000 \text{ gallons.}$$

the lake is not safe because the no. of gallons is above 40 gallons.

Work for problem 2(d)

slope of tangent $= 1 - 3e^{-0.2\sqrt{t}}$

at $t=0$; gallons $= 50 \Rightarrow m_T\big|_{(0,50)} = 1 - 3e^{-0.2\sqrt{0}} = -2$

equation of tangent: $y = -2x + 50$

40 lake is safe $\Rightarrow 40 = -2x + 50$

$\Rightarrow -2x = -10$

$\Rightarrow x = 5$

after 5 days.

GO ON TO THE NEXT PAGE.