AP® Calculus BC
2002 Scoring Commentary
Form B

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Question 1

This problem presented parametric equations for the motion of a particle moving in the *xy*-plane. Part (a) asked for a sketch of the path of the particle. A graphing calculator could have been used as an aid in making this sketch. The purpose of this part was to provide students with a visual reference for the other parts of the problem. For example, the ranges of $x(t)$ and $y(t)$ asked for in part (b) were evident from a correct sketch. Part (c) asked for the smallest value of t at which $x(t)$ was a local maximum. Since $x'(t) = 0$ at this time, the speed of the particle was $|y'(t)| = 2.$ Part (d) asked for a calculation or a suitable estimate of the total distance traveled by the particle. The distinction between total distance traveled and displacement was crucial here, since the total distance traveled was indeed greater than $5\pi,$ but the change in position was only $4\pi.$

The mean score was 5.63.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a), the student lost the first point for an incorrect graph. In part (c), the student calculated the speed incorrectly, losing the third point.

Question 2

This problem presented the rate of change of the amount of pollutant in a lake along with an initial condition and a safety level criterion. Part (a) asked for an interpretation, requiring the calculation of a derivative value with justification. Parts (b) and (c) asked students to find when the pollutant reached its minimum, and whether or not that minimum was below the given safety level. These answers required sufficient justification. Finding the time at which the minimum occurred required solving $P'(t) = 0$, and it was expected that students would use the numerical equation solver of a graphing calculator for this purpose. Justifying that an absolute minimum occurred at a specific time t required an explanation that suitably addressed all values $t \geq 0$. Such a justification could appeal to the sign of the derivative P' for these values. Determining whether the lake was safe, and supporting that determination, was most directly accomplished by evaluating the minimum value by using a definite integral. Part (d) asked for a tangent line approximation to $P(t)$ using the initial condition. While this approximation may have provided reasonable estimates of $P(t)$ for t near 0, in this case students were asked to use the linear model to predict the time at which the pollutant reached a safe level. This prediction was noticeably different from the answer found in part (b).

The mean score was 6.12.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (b), the student lost the point awarded for justification. In part (c), the student reached an incorrect conclusion since the integral was not evaluated correctly.

Question 3

This problem presented a region bounded by two graphs and the *y*-axis. The first two parts involved topics common to Calculus AB and Calculus BC; part (c) was a Calculus BC-only topic. Students were asked to use integration to answer some straightforward questions about this region. Students were expected to use the numerical equation solver of a graphing calculator to find the intersection point of the two graphs. Part (a) required the use of a definite integral to find the area of the region. In part (b), the region was revolved about the *x*-axis, resulting in a solid with cross sections in the shape of "washers," and the volume of the region was asked for, which required another use of a definite integral. Although the integrals could have been evaluated analytically, it was expected that students would use the numerical integration capabilities of a graphing calculator to evaluate these integrals. In part (c), students were asked to write an expression that gave the perimeter of the region. The perimeter was given by the sum of the lengths of two line segments and a definite integral for the arc length of a graph.

The mean score was 7.25.

<u>Sample B</u> (Score 9)

The student earned all 9 points.

<u>Sample D</u> (Score 7)

The student earned 7 points: 3 points in part (a), 3 points in part (b), and 1 point for part (c). In part (c), the student lost the points for the arc length integral, as well as the answer point.

Question 4

In this problem, students were given a graphical representation of a function f and a second function g that was defined in terms of a definite integral of f. The questions asked were most efficiently and directly answered by using the Fundamental Theorem of Calculus and reasoning based on the graph of f. Part (a) asked for calculations of $g(6)$, $g'(6)$, and $g''(6)$. Each value could be found directly using the graph of f. Using the fact that $f = g'$, part (b) required relating the sign of f (positive or negative) to the behavior of g (increasing or decreasing). Similarly, using the fact that $f' = g''$, part (c) required relating the behavior of the slope of the graph of f to the concavity of the graph of g. Finally, part (d) asked for a trapezoidal approximation of a definite integral involving f. All necessary values for the calculation could be obtained from the graph of f and students could take advantage of the symmetry of the graph.

The mean score was 6.72.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 2 points in part (a), 3 points in part (b), 2 points in part (c), and 0 points in part (d). The student miscalculated $g(6)$ in part (a), losing 1 point. In part (d), the student incorrectly used the absolute value of the first and last terms in the calculation.

Question 5

This problem presented a separable differential equation. Part (a) asked for the particular solution to the equation satisfying a horizontal tangent line condition. While no initial value condition was supplied explicitly, students could determine that the point of tangency was $(3, -2)$. Part (a) further asked students to classify the critical point at $x = 3$ as a local minimum, local maximum, or neither. The second derivative test was the most straightforward way to determine this, having used implicit differentiation with the original equation to find an expression for y'' in terms of x and y. Part (b) supplied an explicit initial condition and asked students to solve the separable differential equation. The solution was straightforward, but did require students to choose the appropriately signed square root of a quadratic expression.

The mean score was 5.52.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample D</u> (Score 7)

The student earned 7 points: 1 point in part (a) and 6 points in part (b). The student did not use the first derivative test correctly and arrived at an incorrect conclusion.

Question 6

This problem presented students with an explicit Maclaurin series for $\ln\left(\frac{1}{1-x}\right)$. Part (a) asked for a new Maclaurin series that could be found by algebraic substitution using the given series. The interval of convergence for this new power series could be found using the ratio test. Part (b) presented an alternating series whose sum could be determined by recognizing that it was obtained by substituting $x = -1$ into the given Maclaurin series. Parts (c) and (d) tested students' knowledge of standard examples and tests of convergence of infinite series, including the alternating series test and the so-called "p-series" family of examples. While there was a range of acceptable answers for both parts (c) and (d), the harmonic series was a natural example to use in both cases. It could be utilized by substituting $p = \frac{1}{2}$ for part (c) and $p = 1$ for part (d).

The mean score was 4.22.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample D</u> (Score 7)

The student earned 7 points: 2 points in part (a), 0 points in part (b), 2 points in part (c), and 3 points in part (d). In part (b), the student failed to evaluate the series. In part (c), the student did not give a sufficient reason for the convergence of the series.