AP® Calculus BC
2002 Sample Student Responses

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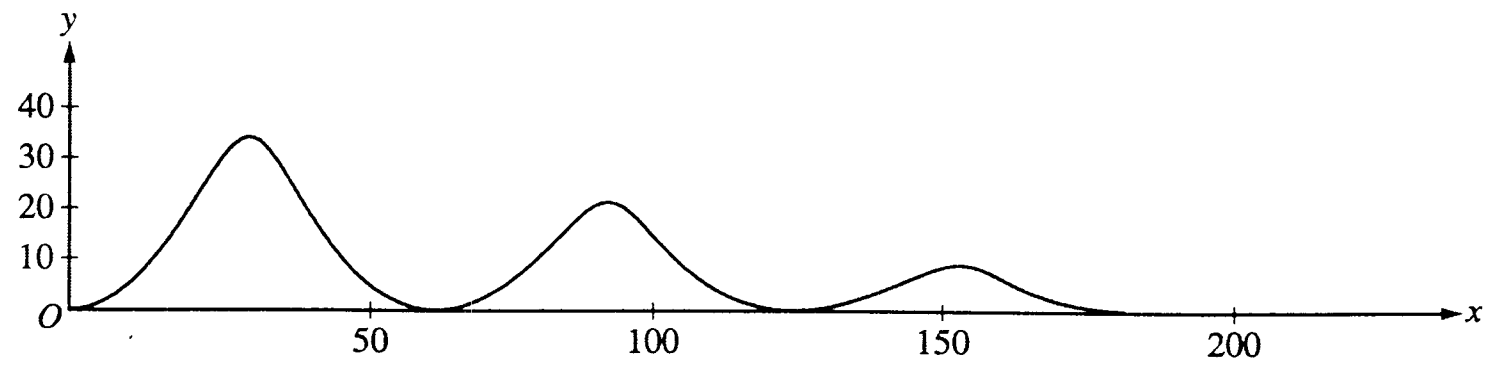
B1



Work for problem 3(a)

$x'(t) = 10 + 4\cos t$

$y'(t) = (20-t)\sin t + \cos t - 1$

$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$ $\frac{y'(2)}{x'(2)}$ = slope

$$\frac{(20-2)\sin(2) + \cos(2) - 1}{10 + 4\cos(2)} = 1.794$$

Work for problem 3(b)

$x(t) = 10t + 4\sin t$

$10t + 4\sin t = 140$

$t = 13.647$ seconds

$x'(t) = 10 + 4\cos t$

$x''(t) = -4\sin t$

$y'(t) = (20-t)\sin t + \cos t - 1$

$y''(t) = -\sin t + (20-t)\cos t - \sin t$

$x''(13.647) = -3.529$

$y''(13.647) = 1.226$

$\overrightarrow{a(t)} = \langle -3.529, 1.226 \rangle$

Work for problem 3(c)

$y(t) = (20-t)(1-\cos t)$

$y'(t) = (20-t)\sin t + \cos t - 1$

$y'(t) = 0$ $\qquad x'(t) = 10 + 4\cos t$

$t = 3.024$ s

$$\text{speed} = \sqrt{(x'(3.024))^2 + (y'(3.024))^2}$$

speed = 6.028 m/s

Work for problem 3(d)

$y(t) = (20-t)(1-\cos t)$

$y(t) = 0$

$t = 6.283$ and 12.566

$x'(t) = 10 + 4\cos t$

$y'(t) = (20-t)(\sin t) + \cos t - 1$

$$\text{Avg speed} = \frac{\int_{6.283}^{12.566} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt}{12.566 - 6.283}$$

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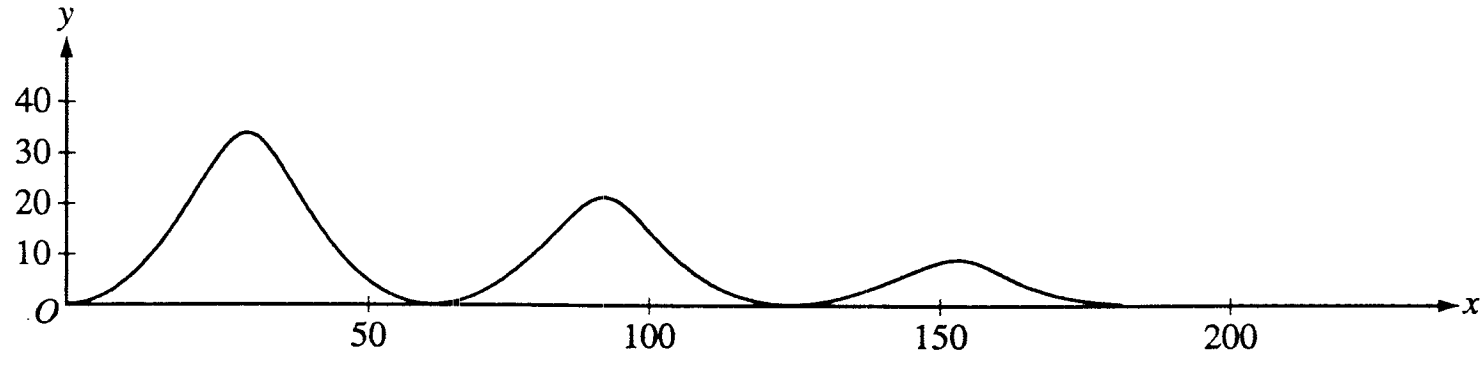
D1



Work for problem 3(a)

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{(20-t)\sin t + \cos t \cdot 1}{10 + 4\cos t}\Bigg|_{t=2} = 1.793$$

Work for problem 3(b)

$a(t) = \langle x''(t), y''(t) \rangle$

$x''(t) = -4\sin t$

$y''(t) = -\sin t + (20-t)\cos t - \sin t$

$= (20-t)\cos t - 2\sin t$

$x(t) = 140 = 10t + 4\sin t$

$t = 13.6471$

$x''(13.6471) \approx -3.529$

$y''(13.6471) \approx 1.2257$

$\vec{a} = \langle -3.529, 1.2257 \rangle$

Work for problem 3(c)

$y'(t) = (20-t)\sin t + \cos t - 1 = 0$

$t = 3.02392$

$x'(3.02392) \approx 6.02766$ m/sec

(don't worry about $\sqrt{x'(t)^2 + y(t)^2}$ because $y'(t) = 0$)

Work for problem 3(d)

$y(t) = (20-t)(1-\cos t) = 0$

$t \neq 20$

$\cos t = 1$

$t = 0, 2\pi, 4\pi, 6\pi$

$$\frac{\sqrt{[x(4\pi)]^2 + [y(4\pi)]^2} - \sqrt{[x(2\pi)]^2 + [y(2\pi)]^2}}{4\pi - 2\pi} = \frac{x(4\pi) - x(2\pi)}{2\pi} \text{ m/sec}$$