AP® Calculus BC
2002 Sample Student Responses

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B1

Work for problem 6(a)

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(2x)^{n+2}}^{\,2x}}{(n+2)} \cdot \frac{n+1}{\cancel{(2x)^{n+1}}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(2x)(n+1)}{n+2}\right| = |2x| \cdot \lim_{n\to\infty}\left|\frac{n+1}{n+2}\right| < 1$$

$$|2x|(1) < 1$$

$$|2x| < 1$$

$$-1 < 2x < 1$$

$$-\frac{1}{2} < x < \frac{1}{2}$$

$-\frac{1}{2}$ $\frac{1}{2}$

$$\boxed{-\frac{1}{2} \le x < \frac{1}{2}}$$

$\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$

$$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty}\left|\frac{1}{n+1}\right| = 0 \quad (ii) \checkmark$$

$$\left|\frac{a_{n+1}}{a_n}\right| < 1$$

$$\left|\frac{\frac{1}{n+2}}{\frac{1}{n+1}}\right| = \left|\frac{n+1}{n+2}\right| < 1 \checkmark$$

converges by the alternating series test

$\displaystyle\sum_{n=0}^{\infty} \frac{(1)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$

$$\int_0^{\infty} \frac{1}{n+1}\,dn$$

$$= \lim_{b\to\infty} \int_0^b \frac{1}{n+1}\,dn$$

$$= \lim_{b\to\infty} \ln|n+1| \Big]_0^b$$

$$= \lim_{b\to\infty} \ln|b+1| - \ln|1|$$

$$= \infty - 0$$

divergent by the integral test

NO CALCULATOR ALLOWED

B_2

Work for problem 6(b)

$$\begin{array}{ccccccccc} & n=0 & & n=1 & & n=2 & & n=3 & \\ f'(x) \approx & 2 & + & \frac{8x}{2} & + & \frac{24x^2}{3} & + & \frac{64x^3}{4} & + \ldots \end{array}$$

$$\boxed{f'(x) \approx \boxed{2 + 4x + 8x^2 + 16x^3 + \ldots + 2^{(n+1)}x^n}}$$

$n=0 \quad n=1 \quad n=2 \quad n=3$

$2^{n+1} \quad 2^2 \quad 2^3 \quad 2^4$

Work for problem 6(c)

$$f'\left(-\tfrac{1}{3}\right) \approx 2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \ldots + 2^{n+1}\left(-\tfrac{1}{3}\right)^n$$

$$\sum_{n=0}^{\infty} 2^{n+1}\left(-\tfrac{1}{3}\right)^n = \sum_{n=0}^{\infty} \frac{2^n \cdot 2 \cdot (-1)^n}{3^n} = \sum_{n=0}^{\infty} 2 \cdot \left(-\frac{2}{3}\right)^n$$

$a = 2$

$r = -\frac{2}{3}$

$$S = \frac{a}{1-r} = \frac{2}{1-\left(-\frac{2}{3}\right)} = \frac{2}{\frac{5}{3}} \cdot \frac{3}{3} = \boxed{\frac{6}{5}}$$

6 6 6 6 6 6 6 6 6 6

NO CALCULATOR ALLOWED

C_1

Work for problem 6(a)

$$\frac{(2x)^{n+1}}{n+1}$$

$$\lim_{n\to\infty} \frac{(2x)^{(n+1)+1}}{(n+1)+1} \cdot \frac{n+1}{(2x)^{n+1}}$$

$$\lim_{n\to\infty} \frac{2x^{\cancel{n+2}\,1}}{n+2} \cdot \frac{n+1}{(2x)^{\cancel{n+1}}}$$

$$\lim_{n\to\infty} \frac{(2x)\cancel{(n+1)}}{\cancel{n+2}}$$

$$-1 < 2x < 1$$

$$-\tfrac{1}{2} \le x < \tfrac{1}{2}$$

$$\sum_{n=0}^{\infty} \frac{2(\frac{1}{2})^{n+1}}{n+1} \qquad \sum_{n=0}^{\infty} \frac{2(-\frac{1}{2})^{n+1}}{n+1}$$

$$\sum_{n=0}^{\infty} \frac{(1)^{n+1}}{n+1} \qquad \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$$

converges when $x = \frac{1}{2}$

① ~~numbers~~ terms decrease
② ~~numbers~~ term $\to 0$
③ signs alternate

converges when $x = -\frac{1}{2}$

$$-\frac{1}{2} \le x \le \frac{1}{2}$$

6 6 6 6 6 6 6 6 6 6

NO CALCULATOR ALLOWED

C_2

Work for problem 6(b)

$$f(x) \approx 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4}$$

$$f'(x) \approx 2 + 4x + 8x^2 + 16x^3$$

$$\boxed{f'(x) = 2 + 4x + 8x^2 + 16x^3 + \ldots + 2^{n+1}x^n + \ldots}$$

Work for problem 6(c)

$$\cancel{f(x) = 2 + \frac{4(2)x}{2} + \frac{8(3)x^2}{3} + \frac{16(4)x^3}{4}}$$

$$\cancel{f'(-\tfrac{1}{3}) = 2 + 4(-\tfrac{1}{3}) + 8(+\tfrac{1}{9}) + 16(-\tfrac{1}{27})}$$

$$\sum_{n=0}^{\infty} 2^{n+1} x^n$$

$$\sum_{n=0}^{\infty} \underline{2^{n+1}}\left(-\frac{1}{3}\right)^n$$