AP® Calculus BC
2003 Sample Student Responses
Form B

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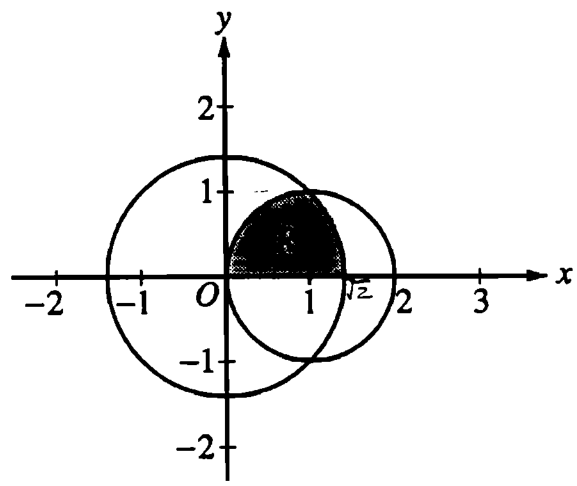
 A,



Work for problem 2(a)

We can say that, $x^2+y^2=2 \Rightarrow y=\sqrt{2-x^2}\ (y>0)$

$(x-1)^2+y^2=1 \Rightarrow y=\sqrt{1-(x-1)^2}=\sqrt{2x-x^2}\ (y>0)$

$$\therefore R=\int_0^1 \sqrt{2x-x^2}\,dx+\int_1^{\sqrt{2}}\sqrt{2-x^2}\,dx$$

(Note that $y=0$ at $x=\sqrt{2}$, in the graph of circle $x^2+y^2=2$.)

Work for problem 2(b)

We can say that $\begin{cases} x^2+y^2=2 \Rightarrow x=\sqrt{2-y^2}\ (x>0) \\ (x-1)^2+y^2=1 \Rightarrow x=1-\sqrt{1-y^2}\ (x<1) \end{cases}$

$$\therefore R=\int_0^1 \sqrt{2-y^2}\,dy-\int_0^1 (1-\sqrt{1-y^2})\,dy=\int_0^1 \{\sqrt{2-y^2}-(1-\sqrt{1-y^2})\}\,dy$$

Continue problem 2 on page 7.

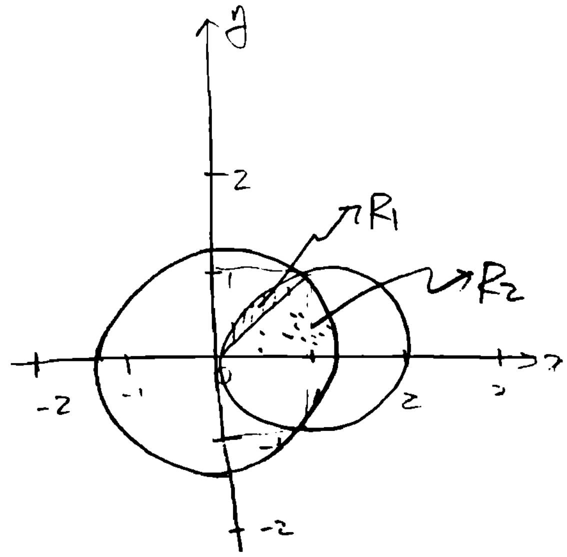
A_2

Work for problem 2(c)

Let $\left(\begin{array}{l} r_1 = \sqrt{2} \\ r_2 = 2\cos\theta \end{array} \right)$

The graph of r_1 r_2 goes through $(1,1)$ when $\theta = \frac{\pi}{4}$ $\left(\because \sqrt{1^2+1^2} = 2\cos\frac{\pi}{4} = \sqrt{2} \right)$

Also, as θ increases from $\frac{\pi}{4}$ to $\frac{\pi}{2}$ on the graph of r_2, r_2 draws the arc of R_2 shown the figure below.



$$\therefore\ S = \int_0^{\frac{\pi}{4}} \frac{1}{2} r_1^2 \, d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} r_2^2 \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\cos^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1+\cos 2\theta) \, d\theta$$

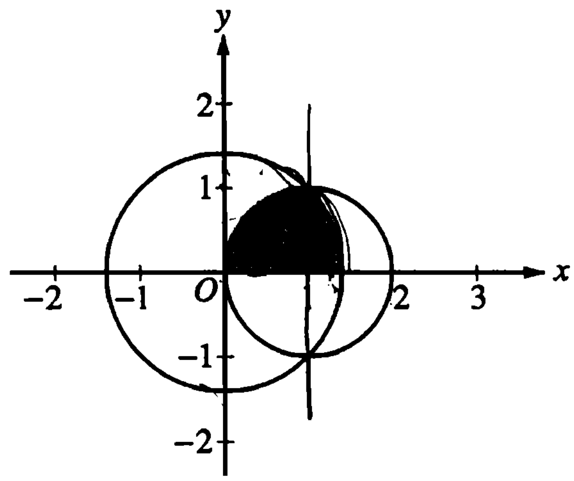
GO ON TO THE NEXT PAGE.

$(x^2+2x+1$

x^2-2x+1

$x = \sqrt{y^2+2x+1}$

$-x^2+2x$



Work for problem 2(a)

$(x-1)^2+y^2=1 \Rightarrow y^2 = 1-(x-1)^2 \Rightarrow y = +\sqrt{1-(x-1)^2}$

$$\int_0^1 \sqrt{1-(x-1)^2}\,dx + \int_1^{\sqrt{2}} \sqrt{2-x}\,dx = 0.785 + 0.368 = 1.153$$

$x^2+y^2=2 \quad y^2 = 2-x$

$y = +\sqrt{2-x}$

Work for problem 2(b)

$x^2+y^2=2$

$\rightarrow x^2 = 2-y^2$

$\rightarrow x = \sqrt{2-y^2}$

$(x-1)^2+y^2=1$

$\rightarrow (x-1)^2 = 1-y^2$

$\rightarrow x-1 = -\sqrt{1-y^2}$

$x = -\sqrt{1-y^2}+1$

$$\int_0^1 \sqrt{2-y^2} - \left(-\sqrt{1-y^2}+1\right)dy = 1.071$$

Continue problem 2 on page 7.

$r \ell_{\frac{1}{2}} l^2 \theta$

Work for problem 2(c)

$S = \frac{1}{2}\int r^2 \theta \quad \frac{1}{2}\int (2\cos\theta)^2 \, d\theta$

$R = \frac{1}{2}\int_{0.817}^{1.5} (2\cos\theta)^2 \, d\theta + \frac{1}{2}\int_{0}^{0.817} (\sqrt{2})^2 \, d\theta$

$= 0.255 + 0.817$

$= 1.072$

GO ON TO THE NEXT PAGE.