AP® Calculus BC
2003 Sample Student Responses
Form B

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Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

Work for problem 3(a)

Since radius $= \frac{1}{2}$(diameter)

$$\Rightarrow \text{Average radius} = \frac{1}{2}\left(\frac{1}{(360-0)\text{mm}}\int_0^{360} B(x)\,dx\right) = \frac{1}{720\text{mm}}\int_0^{360} B(x)\,dx$$

Work for problem 3(b)

$$\int_0^{360} B(x)\,dx = \lim_{\|P\|\to 0}\sum_{k=1}^{3} f(c_k)\,\Delta x$$

$$c_1 = 60\text{mm} \Rightarrow f(c_1) = 30\text{mm}$$
$$c_2 = 180\text{mm} \Rightarrow f(c_2) = 30\text{mm}$$
$$c_3 = 300\text{mm} \Rightarrow f(c_3) = 24\text{mm}$$

$$\Delta x = \frac{360\text{mm}}{n} = \frac{360\text{mm}}{3} = 120\text{mm}$$

$$\Rightarrow \sum_{k=1}^{3} f(c_k)\,\Delta x = 120\text{mm}\left(f(c_1)+f(c_2)+f(c_3)\right)$$
$$= 120\text{mm}\left(30\text{mm} + 30\text{mm} + 24\text{mm}\right) = 10080\text{mm}^2$$

$$\Rightarrow \text{Average radius} = \frac{1}{720}\int_0^{360} B(x)\,dx \approx \frac{1}{720\text{mm}}\left(10080\text{mm}^2\right) = 14\text{mm}$$

Continue problem 3 on page 9.

Work for problem 3(c)

It is the volume of blood in the blood vessel starting from a distance of 125 mm from 1 end to a distance of 275 mm from the same end. The units will be $(mm)^3$

Work for problem 3(d)

$b, a, d, c, e, f \in (0, 360)$
$c > f$, $d > e$, $b > a$

$B''(x) = 0 \Rightarrow \dfrac{B'(b) - B'(a)}{b - a} = 0$

$\Rightarrow B'(b) = B'(a)$

$\Rightarrow \dfrac{B(d) - B(e)}{d - e} = \dfrac{B(c) - B(f)}{c - f}$

Since for all x, Δx is the same

$\Rightarrow B(d) - B(e) = B(c) - B(f)$

From the table there are values of d, e, c, f such that $B(d) - B(e) = B(c) - B(f)$. For example at $x = 300$

$B(360) - B(300) = B(300) - B(240) \Rightarrow 26 - 24 = 26 - 24 \Rightarrow 0 = 0$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

 B_1

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

$F_{avg} = \frac{1}{b-a}\int_a^b F(x)\,dx$

Work for problem 3(a)

$B(x)_{avg} = \frac{1}{360-0}\int_0^{360} \frac{B(x)}{2}\,dx$

$= \frac{1}{360}\int_0^{360} \frac{Bx}{2}\,dx$

Work for problem 3(b)

$\frac{360}{3} = 120$

$B(x)\ avg = \frac{1}{360}\left[\frac{120 f(60)}{2} + \frac{120 f(180)}{2} + \frac{120 \cdot f(300)}{2} \right]$

$= \frac{120}{360}\left[15 + 15 + 12 \right]$

$= \frac{\cancel{12}^{1}}{\cancel{36}_{3}} \times 42$

$= 14$ mm

Continue problem 3 on page 9.

Work for problem 3(c)

$\frac{B(x)}{2}$ = radius of blood vessel

$\pi\int_{125}^{275}\left(\frac{Bx}{2}\right)^2 dx$ Volume of the blood vessel from $x = 125$ mm to $x = 275$ mm in $(mm)^3$

Work for problem 3(d)

At x where $B''(x) = 0$

There is an inflection on the graph

The sign of $B'(x)$ changes

$B'(x)$ = the change of diameter

From the table we know that when the diameter increases $B'(x) > 0$ when diameter decrease $B'(x) <$

$B'(x)$ changes signs

$\therefore B''(x) = 0$

END OF PART A OF SECTION II

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