AP® Calculus BC
2003 Sample Student Responses
Form B

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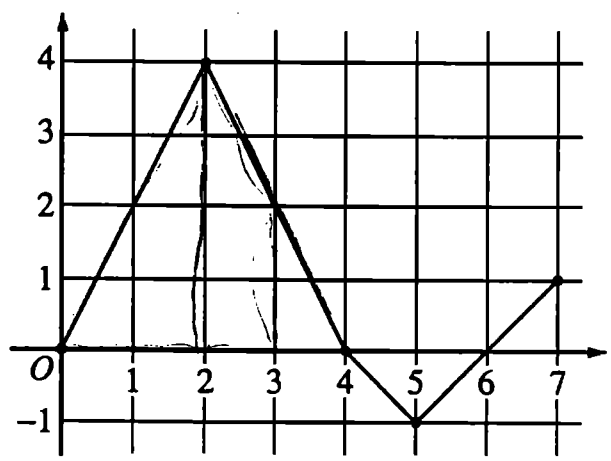
NO CALCULATOR ALLOWED



Graph of f

Work for problem 5(a)

4+3

$g(x)=\int_2^x f(t)\,dt$

$g'(x)=f(x)$

$g''(x)=f'(x)$

$g(3)=\int_2^3 f(t)\,dt = F(3)-F(2)=\pi-4=3$

$g'(3)=f(3)=2$

$g''(3)=f'(3)=-2$

$g(3)=3$
$g'(3)=2$
$g''(3)=-2$

Work for problem 5(b)

rate of change of $g = g'(x)$

$$\frac{1}{3-0}\int_0^3 g'(x)\,dx = \frac{1}{3}\{g(3)-g(0)\}$$
$$= \frac{1}{3}\{3-g(0)\}$$
$$= \frac{1}{3}\left(3-\int_2^0 f(t)\,dt\right)$$
$$= \frac{1}{3}\left(3+\int_0^2 f(t)\,dt\right)$$
$$= \frac{1}{3}\left(3+F(2)-F(0)\right)$$
$$= \frac{1}{3}(3+4-0)$$
$$= \frac{7}{3}$$

$\frac{7}{3} \simeq 2.333$

Continue problem 5 on page 13.

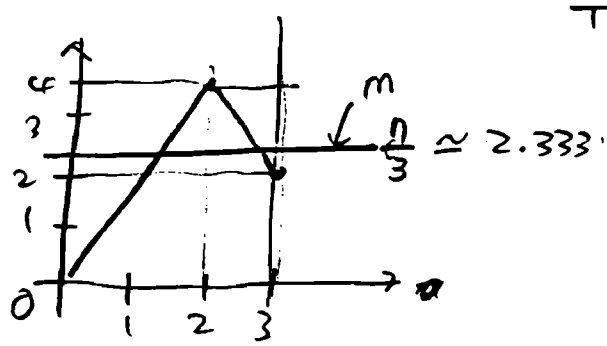
NO CALCULATOR ALLOWED

Work for problem 5(c)

$g'(c) = \frac{\pi}{3}$

since $g'(x) = f(x)$, $g'(c) = f(c)$.

$g'(c) = f(c) = \frac{\pi}{3}$



The line m crosses the graph of f twice

→ $g'(c)$ is equal to 2.333 at two values of c.

Work for problem 5(d)

At points of inflection, $g''(x)$ should change from (+) to (−), or vice versa.

At $f(2)$, $f'(x)$ changes from (+) to (−), and at $f(5)$, $f'(x)$ changes from (−) to (+).

Points of inflection exist at $x=2$ and $x=5$.

B,

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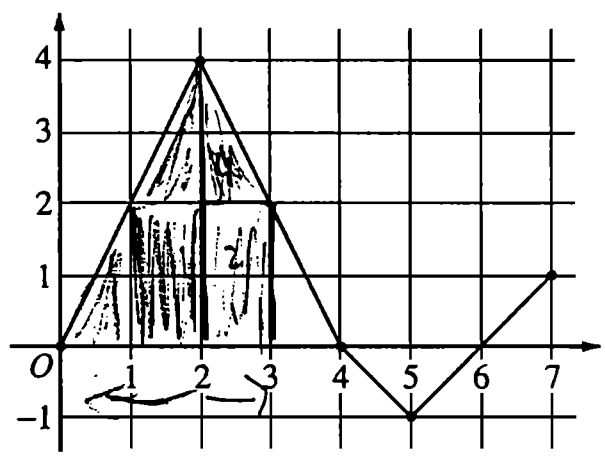

Graph of f

Work for problem 5(a)

$$g(3) = \int_2^3 f(t)\,dt = \boxed{3}$$

$f(x)=$ $\Rightarrow g'(3) = f(3) = \boxed{2}$

$f(x)$

$g''(3) = f'(3) =$ slope at $3 = \frac{2-4}{3-2} = \frac{-2}{-1} = \boxed{2}$

Work for problem 5(b)

avg rate of change $= \frac{g(a)-g(b)}{a-b}$

$g(0) = \int_2^0 f(t) = -4$

$g(3) = \int_2^3 f(t) = 3$

$$\frac{g(0)-g(3)}{0-3} = \frac{-4-3}{-3} = \boxed{\frac{7}{3}}$$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$g'(c) = 7/3 \Rightarrow$

$f(c) = 7/3$ at 1 (one) point

because

on $(0,2)$, $f(x) = y = 2x$

$2x = 7/3$

$x = 7/6 \leftarrow$ only at $x = 7/6$

on $(2,3)$, $f(x) = y = 2x + 8$

$7/3 = 2x + 8$

$7/3 - \frac{24}{3} = 2x \Rightarrow -\frac{17}{3} = 2x$ ~~$x = -17/6$~~ (not on $(0,3)$)

Work for problem 5(d)

point of inflection $= g''(x) = 0$

$g''(x) = f'(x)$

$f'(x) = 0$ at $x = 2$, $x = 5$

$f'(x)$ + + + + | − − − − − | + + + + +

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